Labwork in photonics.

Polarization.

Engineer - 2^{nd} year - S7 -Palaiseau
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Polarization : "Well begun is half-done [1]"

The purpose of polarization labs is to illustrate recurring experimental situations where you will be brought to understand the effect of polarization on the propagation of a light wave. They use some of the concepts covered during the first year course and it is important to read it again before starting the labs.

The educational objectives of the four sessions are summarized below. Before the first lab, you must know:

- the definitions of linear and circular birefringence,
- the effects of a half-wave plate and a quarter-wave plate on a given polarization state,
- the polarization state exiting a birefringent plate illuminated by a linear polarization state at 45° of its neutral axes,
- the influence of the wavelength on the birefringence properties of a material,
- the definition of Brewster's angle.

At the end of the session, you should know:

- how to produce a given polarization state,
- how to analyze a given polarization state, using several methods,
- how to measure a linear birefringence, using several methods,
- how to characterize a medium having a circular birefringence,
- how to make an amplitude modulation with electro-optic materials.

[1] Mary Poppins, 1964

The following few basic questions should help you prepare the polarization labs. **You must have answered these questions before you start your first polarization lab session.** The answers can be found in your polarization lecture notes and/or in the appendices.

The written answers will be graded out of 10 and given to the teacher **at the beginning of the first polarization lab session**, regardless of the lab by which you start.

Furthermore, you must prepare each session by reading the booklet and answering the questions of preparation. Instructions are given at the beginning of each subject.

P1 What is the effect of a polarizer on light?

P2 What is meant by saying that a material is "birefringent"?

P3 What is the operating principle of a wave plate? What is the definition of a neutral axis?

P4 What is the phase shift introduced by a $\lambda/4$ plate (quarter-wave plate or QWP)? Same question with a $\lambda/2$ plate (half-wave plate or HWP).

P5 What is the effect of these plates on an incident linear polarization state?

P6 Do the properties of a wave plate depend on wavelength? If yes, how so?

P7 What is the "ellipticity" of polarized light?

P8 What is the polarization exiting a wave plate when it is illuminated by a linearly polarized light at $45°$ from the neutral axes of the plate?

P9 How are "transverse electric" (TE) and "transverse magnetic" (TM) polarizations defined? What is Brewster's angle?

Labwork 1

Basic polarization experiments

If this is your first polarization lab session, do not forget to prepare the questions page 4 (graded out of 10, they should be handed to the teacher at the beginning of the session).

The goal of the first session is the study of various phenomena of polarization related to the birefringence of materials. The birefringence is linear (in the case of wave plates) or circular (in the case of optically active materials). It is important to make the difference between these two phenomena.

The first part is a review of the basics of your first year course. Prepare questions in advance, and check your observations match what you expected.

Parts 2.1 and 2.2 will lead to an oral presentation graded out of 5 points. You must have thought about the theoretical issues of these parts before the session, and have prepared schemes to explain the method of analysis of a polarized state.

Contents

Equipement:

- A polariscope (see figure 1.1) : wooden structure equipped with a dichroic polarizer (bottom stage), a graduated dichroic analyzer (top stage) and two intermediate stages with mounts that allow rotations of exactly 45°. You can use the polariscope with either monochromatic or white light. To change the light source, just rotate the polariscope.
- A set-up on a bench, using a mercury lamp with a green filter or white light.
- A fibered spectrometer connected by USB to the computer.
- Samples and wave plates: **handle them with special care and place them back in their boxes after use.**
- A setup to observe interferences with white convergent light.

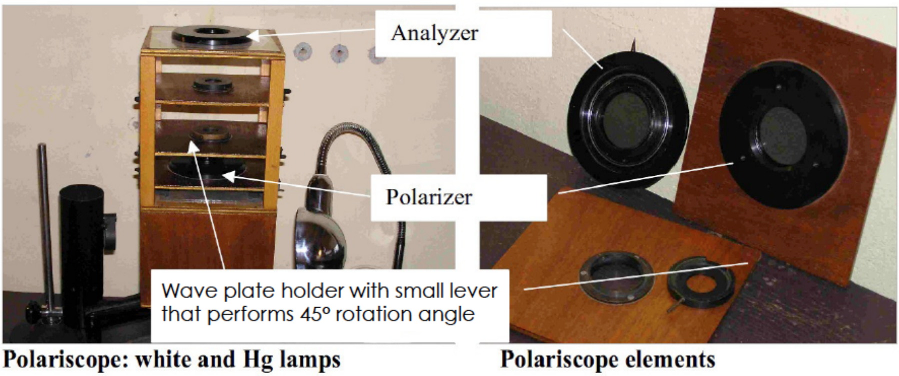


Figure 1.1: The different components of a polariscope.

1 Study of polarizers and wave plates with the polariscope

In the first part you will study the properties of some dichroic polarizers and wave plates. For this purpose you will shine monochromatic light on the polariscope. You should be able to go over this introductory part quickly, especially if you have already done the other polarization lab sessions.

Maximum time to spend on this first part: 45 minutes.

1.1 Find the axis of a polarizer using reflection at Brewster's angle

At Brewster's angle, the TM polarization (the one parallel to the incidence plane) is not reflected at all. The intensity of the light reflected by a glass plate (or by the linoleum of the floor) will therefore go to zero when analyzed through a particular direction of the axis of a polarizer. This property is used in photography for filters or anti-reflex polarizers.

P1 Write the formula for Brewster's angle for an air-glass reflection. Calculate this angle for an ordinary glass of refractive index $n = 1.5$.

In order to find Brewster's angle with a good accuracy, you can illuminate a glass plate fixed on the table with a well oriented desk lamp.

⇝ Practice orienting the axes of several polarizers using Brewster's angle. With this method, orient the analyzer of the polariscope (using the rotating ring adjustable from below) so that the direction of the needle corresponds to the direction of the transmitted polarization. In the following part, the tip will indicate the absolute direction of the axis of the polariscope.

For convenience, you can place the top polarizer (analyzer) so that its axis corresponds to the graduation 0 of the mount.

1.2 Study of half- and quarter-wave plates

For this part, you will use the polariscope with a mercury lamp with a green filter to select its green line.
You can use different kinds of wave plates: mica wave plates, polymer wave plates (made of plastic), quartz wave plates (the small ones). Unless otherwise specified in the text, use the plastic plates that are less fragile and less expensive than the others.

Find the neutral axes of the wave plates

⇝ Cross the analyzer and the polarizer axes in the absence of the wave plate under study. Then place the wave plate between polarizer and analyzer and rotate it in order to reach minimum light transmission without rotating the analyzer or the polarizer.

Q1 Explain in a simple way why this method allows you to find the directions of the neutral axes of the wave plate. Is the direction perpendicular to a neutral axis, a neutral axis as well?

Half-wave plate

⇝ Place a half-wave plate (HWP) designed for the green line of the Hg lamp (546.1 nm) between crossed polarizer and analyzer. Put one of the neutral axes of the plate along the direction of the incident polarization (imposed by the direction of the polarizer). This orientation of the wave plate will be taken as a reference. Rotate the plate by an arbitrary angle θ with respect to your reference and then rotate the analyzer.

Q2 Can you find the extinction again? What kind of polarization exits the HWP?

⇝ Rotate the plate by $\theta = 45^{\circ}$ using the mount that allows **rotations of precisely 45°**.

Q3 Determine the output polarization. Is it the expected result?

Quarter-wave plate

⇝ Replace the HWP by a quarter-wave plate (QWP) designed for the green line of the Hg lamp. Find the neutral axes of this plate. Using the calibrated rotating mount, adjust the neutral axes of the QWP at $\theta = 45^{\circ}$ from the direction of the polarizer.

Q4 What do you observe when you turn the analyzer? What kind of polarization exists the QWP? Explain your observation using a few sentences and sketches.

⇝ Now turn the QWP by half the calibrated angle (about 20°). Note for which directions of the analyzer the intensity of the transmitted light reaches a maximum and a minimum.

Q5 Verify that the obtained polarization is elliptical and compare with the expected direction of the major axis of the ellipse.

2 Production and Analysis of a polarization state

For this part of the tutorial you have at your disposal an optical bench, two lenses, two polarizers and two quarter-wave plates (the mounts of the two plates are graduated and the 0 indicates the direction of the slow neutral axis). You have to find how to produce and analyze a given state of polarization.

For a better accuracy of your measurements, it is strongly recommended to verify the exact orientation of the neutral axes of the quarter wave plates and the axis of the polarizers. This is probably not exactly 0!

Q6 There are two light sources: a mercury-vapor lamp with a filter to select the green line and a white lamp. Which one will you use? Why?

Q7 Birefringent plates are designed to be used at normal incidence. What happens if this condition is not fulfilled?

⇝ Pay particular attention to fulfill this condition when manipulating.

2.1 Production of a polarization state

We first want to produce a right-handed elliptical polarization with an ellipticity equal to 30°.

Q8 Keeping first aside the handedness of the polarization, explain how to place the neutral axes of a quarter-wave plate in order to produce an elliptical polarization of a given ellipticity. Draw a sketch.

Q9 On the previous sketch, place the slow axis of the plate so that the produced polarization is right-handed. In order to do so, you can ask yourself which component of the electric field has its phase in advance of the other.

⇝ Perform this configuration experimentally on the optical bench.

2.2 Analysis of a polarization state

The goal is now to analyze the polarization state that you have produced. You should thus consider the incident polarization state as unknown (i.e. the setup achieved at the previous question is now a “black box”).

The first step is to check whether the polarization state is linear or more complex.

⇝ Form the image of the source on a screen and add a polarizer on the light path. Observe what happens when you rotate the polarizer.

Q10 What can be expected in general, depending on the different types of polarization states?

In our case, the vibration is totally polarized (there is no unpolarized component). The idea of the analysis is then to transform the polarization state into a linear polarization which will be easy to characterize with an analyzer.

⇝ Place the previous polarizer (used as an analyzer) in the direction such that the intensity is minimum, then place a quarter-wave plate before, so that one of its neutral axes is aligned with the axis of the analyzer.

Q11 What kind of polarization exits the QWP? Why? What are the possible directions of this polarization?

Q12 Choosing a handedness of the incident polarization state, make two sketches corresponding to the two possibilities for positioning the slow axis of the quarter-wave plate. Give the orientation of the linear polarization exiting the plate in each case.

Q13 Following on the example that you chose previously, by which angle and in which direction should you turn the analyzer in order to reach an extinction in each case (depending on the two possible orientations of the quarter-wave plate)? Deduce how the slow axis of the quarter-wave plate should be initially oriented with respect to the analyzer so that the direction and angle of rotation of the analyzer gives exactly the handedness and ellipticity of the polarization state.

Q14 Describe, step by step, a **general** procedure to analyze a polarization state from the above manipulations and apply it to the vibration produced in Sec. 2.1 as a validation.

Present it to your lab supervisor, using a few sketches. This presentation is graded on 5.

2.3 Analysis of an unknown polarization device

In this part you will use the device called "polariseur photo". It will replace the previous setup producing the polarization state.

⇝ Analyze the polarization state exiting the device, depending on the incident polarization state, and for both directions of propagation of light through the device.

Q15 Guess what this "polariseur photo" is made of, from your observations.
(Bonus question: this type of polarizing filter is used in photography or in 3D movie glasses, do you see the purpose in each case?)

3 Optical activity and circular birefringence

In this part of the lab session you will study a birefringent material where light propagates along its optical axis.

⇝ Use a thin quartz plate ($L = 7.7$ mm) cut perpendicularly to the optical axis. Align carefully the following setup (align precisely the elements along the bench, adjust the position of the lenses using autocollimation, etc) to illuminate the sample with a parallel beam from the green line of Hg lamp. Ask the teacher to check your setup.

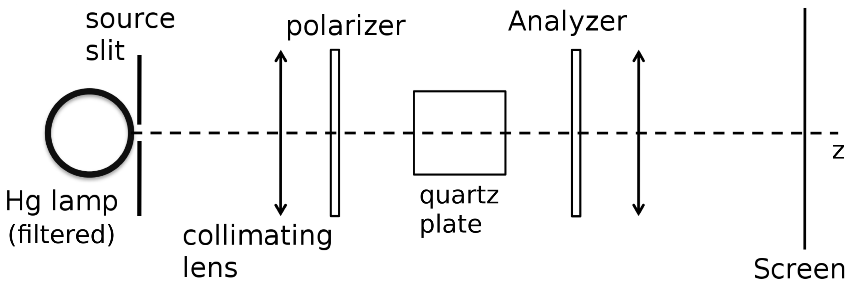


Figure 1.2: Experimental setup.

⇝ Analyze the polarisation state exiting this sample.

Q16 What happens when you turn the plate around the z axis of the setup? How is the outcoming polarization changed when you turn the incident polarization by an angle θ? Coming back to a configuration where the polarizer and analyzer are crossed, by what angle should you turn the analyzer in order to find extinction in the presence of the quartz plate? What is the effect of the quartz sample on a linear polarization?

⇝ Using the appropriate equipment, illuminate the quartz plate with a left-handed circular polarization. What is the outcoming polarization? Did the handedness change? Now answer the same questions with an incident polarization that is rigth-handed circular.

Q17 What is the basis of natural decomposition of the polarization in this case?

You can understand this property as a circular birefringence effect, in which right and left circular polarizations propagate in the medium with two different refraction indices, n_r and n_l, and thus acquire a phase difference:

$$\varphi = \frac{2\pi}{\lambda}(n_r - n_l)L = 2\alpha.$$

We can write $\alpha = \rho L$ where L is the sample thickness and ρ a characteristic of the active medium, inversely proportional to the wavelength squared, following the (approximate) Biot law:

$$\rho = \frac{A}{\lambda^2}.$$

Q18 The specific rotation of quartz is $\rho = 22.09°.mm^{-1}$ at $\lambda = 589.3$ nm. Is that compatible with your measurement in question **Q16**? Estimate the sources of uncertainty.

⇝ Illuminate the plate with white light.

Q19 Explain why you observe colors on the screen. What happens when you rotate the analyzer?

⇝ Observe the spectrum of the light exiting the system with a spectrometer and confirm your previous observations.

Q20 Observe what happens on the spectrum when you rotate the analyzer: can you deduce from this observation whether the quartz plate is left or right-rotatory?

When the sample with rotatory power is thicker, several wavelengths of the visible spectrum are simultaneously cut. The exiting color is a high-order white, that can be analyzed with the spectrometer.

⇝ Put now the long quartz sample, cut perpendicularly to the optical axis, on the set-up. Its length is $L = 60.10 \pm 0.01$ mm (as indicated on the mount). Observe the spectrum obtained between crossed polarizer and analyzer.

Q21 Explain the origin of the dark fringes.

Q22 Determine, using the method you have explained at **Q20**, if the quartz is left- or right- rotatory.

Q23 Calculate the angle α by which the polarization state rotates in the sample for $\lambda = 589.3$ nm (yellow).

⇝ For which orientation of the analyzer do you reach an extinction at this exact wavelength?

Q24 Check the consistency with the calculation. If it is not, check the alignment of the crystal on the set-up. Why is it critical?

⇝ Between crossed polarizers, write down the cut wavelengths.

Q25 How can you determine the rotation angle for all these wavelengths, using the calibration wavelength $\lambda = 589.3$ nm?

Q26 Using the previous measurements, plot the curve $\alpha = f(1/\lambda^2)$. Does the sample rotatory power follow Biot's law?

Q27 Deduce from the curve the value of A for the studied quartz. Compare this value with the calibration value at $589,3$ nm.

4 To go further

This section provides more complex examples than those illustrated above, but also encountered much less frequently. Do this part only if you have done and understood all the previous questions. Otherwise, it is better to spend more time to master the basic concepts. This part is not mandatory in the report.

The polarization states exiting a birefringent plate can be predicted fairly well if the plate is illuminated at normal incidence and at a given wavelength. This exercise is more difficult if the light is polychromatic or if the beam is not collimated. The following examples will help you to understand a little better the phenomena under these conditions.

4.1 White light observations with HWP (at 546.1 nm)

The characteristics of the wave plates that you studied in the first part depend on the wavelength.

⇝ Rotate the polariscope in order to illuminate it with white light (use the desk lamp). Cross the analyzer and the polarizer and place a HWP (at 546.1 nm) in between. Adjust the neutral axes of the studied plate at 45° from the direction of the polarizer. Then turn the analyzer to try to recover the extinction.

Can you reach the extinction in this case? What color do you observe if the analyzer and the polarizer are crossed? parallel?

When rotating the plate around one of its neutral axes, the optical path difference increases or decreases (depending if the rotation is around the ordinary or extraordinary axes). Draw a sketch in one of those cases to explain the variation (increase or decrease) of the optical path difference.

⇝ Observe the color variations when you rotate the plate around each of its neutral axes between crossed and parallel polarizers. Carefully write down your observations.

Use Newton's color scale (given in appendix pg.14) to check that the studied plate is a half-wave plate at 546 nm. What is the optical path difference introduced by this plate?

4.2 Obtaining the slow axis of a birefringent plate

Here we use a set-up that allows us to illuminate convergent white light on a spar crystal (negative uniaxial crystal) that is cut perpendicularly to the optical axis, and placed between crossed polarizers.

- Describe the interference pattern that you observe. What happens when the crystal is rotated around its optical axis? What happens when you turn the polarizer and analyzer while keeping them crossed?

- Deduce the origin of the black cross. Explain how the interference pattern is formed. In particular, explain why one finds Newton's color scale as one moves away from the white (or black) center of the pattern.

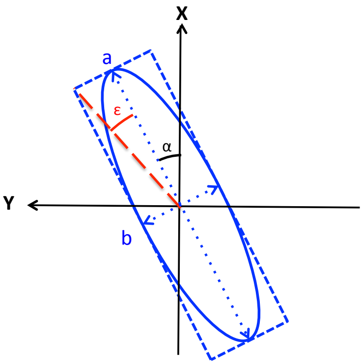
- Place a QWP used to analyze a polarization state in the path of the light so that its neutral axis is at 45° from the black cross. Explain why two black spots (corresponding to zero path-difference) appear on the interference pattern in the direction of the slow axis.

5 Appendix : Newton's color scale

δ in nanometers optical path difference	scale with white center $I = I_0 \cos\left(\frac{\pi\delta}{\lambda}\right)$	scale with black center $I = I_0 \sin\left(\frac{\pi\delta}{\lambda}\right)$
0	white	black
40	white	iron-gray
97	yellowish-white	lavander-gray
158	yellowish-white	grayish-blue
218	brown yellow	clear gray
234	brown	greenish white
259	light red	almost pure white
267	carmin red	yellowish-white
275	dark brownish-red	pale straw-yellow
281	dark violet	straw-yellow
306	indigo	light yellow
332	blue	bright yellow
430	greyish-blue	brownish-yellow
505	bluish-green	reddish-orange
536	light green	red
551	yellowish-green	deep red
565	light green	purple
575	greenish-yellow	violet
589	golden yellow	indigo
664	orange	sky blue
728	brownish-orange	greenish-blue
747	light carmin red	green
826	purple	light green
843	violet purple	yellowish-green
866	violet	greenish-yellow
910	indigo	pure yellow
948	dark blue	orange
998	greenish-blue	bright reddish-orange
1101	green	dark violet red
1128	yellowish-green	light bluish-violet
1151	dirty yellow	indigo
1258	skin color	blue (greenish tint)
1334	brownish-red	sea green
1376	violet	bright green
1426	greyish violet blue	greenish-yellow
1495	greenish-blue	pink (light tint)
1534	blue green	carmin red
1621	pale green	carmin purple
1658	yellowish-green	violet grey
1682	greenish-yellow	greyish-blue
1711	greyish-yellow	sea green
1744	greyish-red mauve	bluish-green
1811	carmin	nice green
1927	reddish-grey	gris green
2007	greyish-blue	almost white grey
2048	green	light red
2338	light pink	light blue green
2668	light blue green	light pink

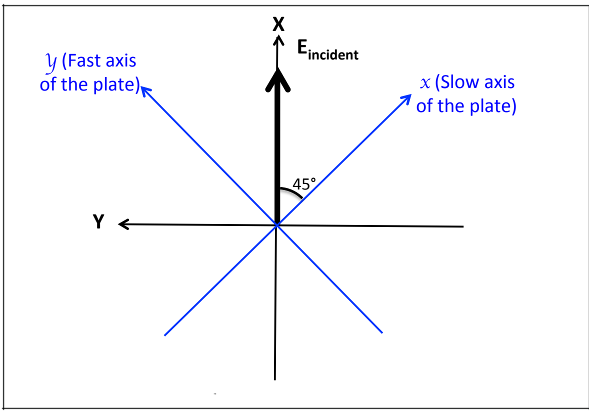
6 Appendix: Measurement of a wave plate birefringence by measuring the exiting ellipticity

Parameters describing an elliptic polarization



α : direction of the major axis of the ellipse
a : dimension of the major axis of the ellipse
b : dimension of the minor axis of the ellipse
$|\tan\varepsilon| = b/a$: ε : ellipticity of the ellipse + handedness

Let us place a wave plate having its neutral axes at 45° from the incident linear polarisation. The exiting polarisation state is elliptic:



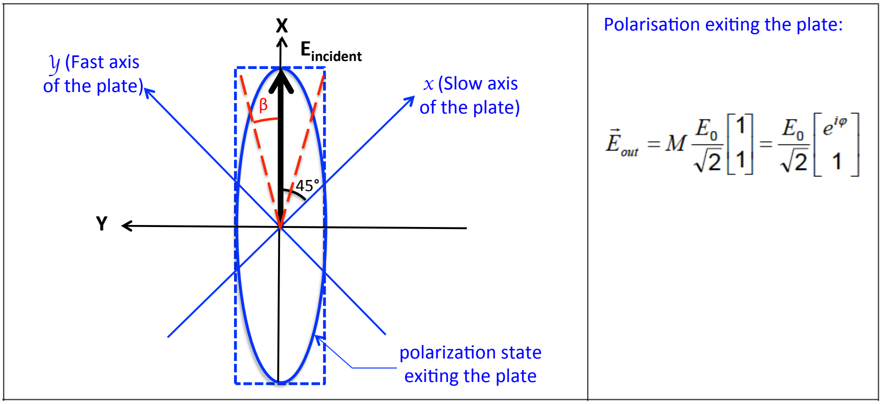
Linear polarization at 45° entering the wave plate

Incident Jones' vector:

$$\vec{E}_{in} \frac{E_0}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$$

Jones' matrix for the wave plate, introducing a phase shift ϕ between the two eigen polarization states:

$$M = \begin{bmatrix}e^{i\varphi} & 0\\0 & 1\end{bmatrix}$$



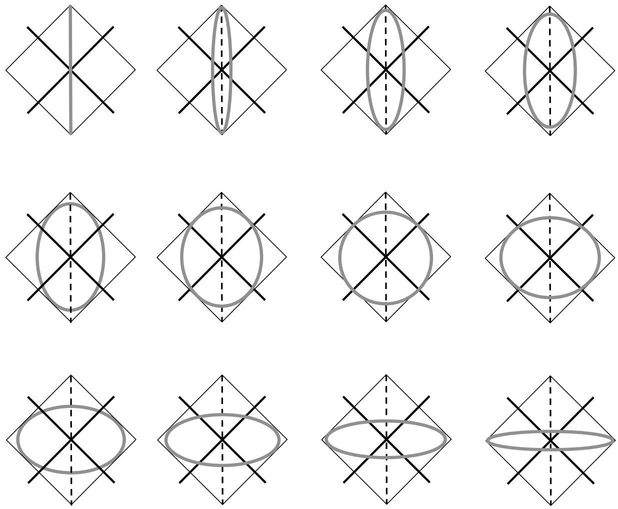
In the reference system (X, Y) of the incident polarization:

$$\vec{E}'_{out} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \vec{E}_{out} = \frac{E_0}{2} \begin{bmatrix} 1 + e^{i\varphi} \\ 1 - e^{i\varphi} \end{bmatrix} = E_0 \begin{bmatrix} \cos(\varphi/2) \\ \sin(\varphi/2) \end{bmatrix} \tag{1.1}$$

This Jones' vector corresponds to an elliptic polarization state whose axes are parallel and perpendicular to the direction of the incident polarization. Let us note β the angle between a diagonale of the rectangle within which is inscribed the elllipse described by the end of the electric field vector of this elliptic polarization state and the X axis of the linear incident polarization (see scheme above). β follows this relationship:

$$\tan|\beta| = \left|\tan\left(\frac{\varphi}{2}\right)\right| \; ; \beta < \pi/2$$

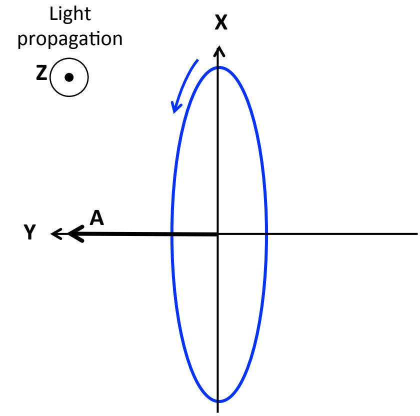
According to the previous scheme, β is strictly equivament to the ellipticity ε. But, be careful! You can find cases where $\beta > 45°$. In these cases, the ellipse major axis is perpendicular to the incident polarization direction. The relationship $\tan|\beta| = \left|\tan\left(\frac{\varphi}{2}\right)\right|$ still stands but $\varepsilon = 90° - \beta$. To measure φ, β has to be measured. And the ellipticity of the exiting polarization state can also be deduced from β. On the following figure, the phase shift introduced by the plate goes from 0 to 180° by steps of 15°. The incident polarization state (grey bold line, for a zero phase shift, or grey dashed line) is at 45° from the neutral axes of the plate (black solid lines). The ellipticity of the exiting polarization is such that the ellipse in inscribed inside a square ($E_{Ox} = E_{Oy}$).



- The axes of the ellipse have a fixed orientation, at 45°.
- The ellipticity ϵ is equal to $\pm\varphi/2$ (modulo $\pi/2$)

Ellipticity measurement using a quarter-wave plate This method has two steps:

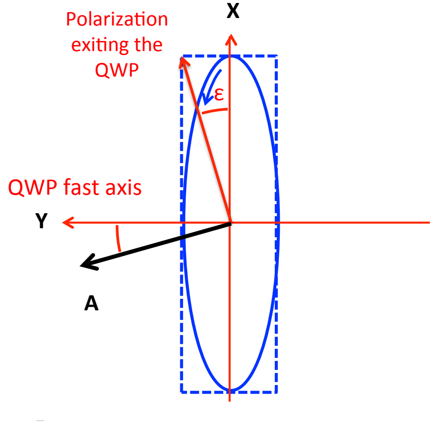
1. Determine the direction of the small axis of the ellipse with an analyzer (minimum reached).
2. Add a quarter wave plate having its slow axis perpendicular to the small axis of the previous ellipse. A linear polarization state having an angle ε with respect to the slow axis of the QWP is now exiting the QWP. The extinction is reached again by turning the analyzer of ε. In that case, the angle of rotation of the analyzer is less than or equal to 45°.



Left-handed elliptic polarization with an ellipticity ε so that tan ε = *b/a* with

$$\vec{E} = \begin{bmatrix} a \\ ib \end{bmatrix}$$

The minor axis is located using an analyzer. Then, a QWP is introduced before the analyzer.



The QWP produces a linear polarization:

$$\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix} \begin{bmatrix} a \\ ib \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

The analyzer is rotated by an angle ε to reach extinction again.

Labwork 2

Birefringence measurements

If this is your first polarization lab session, do not forget to prepare the preliminary questions page 4 (graded out of 10, they should be handed to the teacher at the beginning of the session).

Contents

The objective of this session is to determine as accurately as possible the birefringence of some crystalline plates (that is to say the path difference (OPD) introduced between the two neutral axes) using different methods. Depending on the samples, some methods are applicable or not, more or less accurate, complementary... In any case, it is essential to exploit all the results on site to immediately detect any incompatibilities between the measurements, and **make all the necessary verifications on site**. We will try, for each method, to estimate the uncertainties and to minimize them.

It is essential that each student should make all measurements by all methods. The order of measurement methods is not very important.

The samples to be studied are quartz plates, placed in the box Polarization B.

These samples are really fragile and expensive (about 300 euros P.U.). Therefore, you must handle them with care and put them back in their boxes after use.

At the end of the session, you will present the principle of the three measurement methods. This presentation is **graded out of 5**. To prepare it properly, you must answer the questions **P1** et **P2** (the annex 6 of lab 1 will be helpful) before the session.

Methods used:

- Observation of a channeled spectrum with a USB grating spectrometer
- Measurement of the OPD with a Babinet compensator
- the so-called "Quarter-wave plate method": analysis, on a polariscope illuminated with a monochromatic light, of the light vibration exiting the plate under study, when the input polarization is oriented at an angle of 45° with respect to the wave plate axes.

During the lab session, you will summarize all the obtained results, for each plate, in a table. You will also draw $\delta(\lambda)$, the OPD as a function of λ for all the measurements and you will verify the obtained trend and the consistency of your results.

1 Preparation

P1 What kind of polarization state exits a birefringent plate when the incident polarization is linear and oriented at $45°$ from the neutral axes of the plate? Precise the orientation of the exiting polarization state. Draw a scheme showing the incident polarization, the neutral axes of the plate, and the exiting polarization state. We keep aside the handedness of the exiting polarization.

P2 Give the relationship between the ellipticity ε of the exiting polarization and the phase shift φ introduced by the plate.

2 Study of the dark fringes in the white light spectrum

$\rightsquigarrow$ Carefully align the setup. That means carefully align all the optical components on the optical bench, fix the lenses at the correct position with the auto-collimation method, in order to shine collimated light on the sample. This setup is the same as the one used during the tutorial Polarization 1 to study the quartz rotatory power.

⇝ Orient the polarizer at 45° with respect to the vertical axis, and cross precisely the analyzer with it.

⇝ Place the plate under study on the bench and orient its axes at 45° to those of the polarizer and the analyzer.

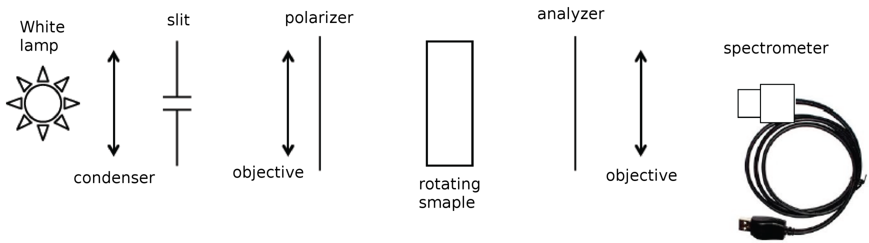


Figure 2.1: Experimental setup

Q1 Explain clearly and simply the presence of dark fringes in the spectrum between parallel polarizers and between crossed polarizers.

Q2 What is the effect on the spectrum of a sample rotation around the optical axis of the set-up?

The positions of the dark fringes **between polarizer and analyzer either crossed or parallel** can be traced back to the value of the optical path difference for specific wavelength values.

The contrast of the interference fringes is maximal if the neutral axes of the plate are at 45° of parallel or crossed polarizers.

For crossed polarizers, the intensity at the output of the analyzer is:

$$I = I_0 \sin^2\left(\frac{\varphi}{2}\right) = I_0 \sin^2\left(\frac{\pi\delta}{\lambda}\right)$$

You can see the extinction of all wavelengths for which the optical path difference introduced by the plate is an integer multiple of the wavelength, ie $\delta = k\lambda_k = (n_e - n_o)e$.

For parallel polarizers,

$$I = I_0 \cos^2\left(\frac{\varphi}{2}\right) = I_0 \cos^2\left(\frac{\pi\delta}{\lambda}\right)$$

You can see the extinction of all wavelengths for which the optical path difference introduced by the plate is a half integer multiple of the wavelength, ie $\delta = (k + 1/2)\lambda_{k+1/2} = (n_e - n_o)e$.

The measurement of the wavelength of two successive dark fringes (corresponding to k and $k + 1$) or of two dark and bright successive fringes (corresponding to k and $k + 1/2$) allows in principle to determine simply the value of k (by solving an equation with one unknown). But beware, the variation of birefringence with wavelength, even if it is small, sometimes makes this determination difficult: we never find an integer value of k! Remember this and use the fact that the birefringence $n_e - n_o$ decreases with increasing wavelength (Cauchy's law) to determine k (by solving an inequality with one unknown).

Practical method to check each studied plate:

- Enter into an Excel spreadsheet, in ascending or descending order, all the measured values of the wavelengths corresponding to the dark fringes between crossed and parallel polarizers.
- Then determine the value of k**, positive integer,** for each dark fringe.
- Calculate the optical path difference for each dark fringe and plot the OPD as a function of wavelength: $\delta(\lambda) = k\lambda_k$ or $(k + 1/2)\lambda$.
- Check the consistency of your measurements and calculations, in particular the expected decrease of the OPD with the wavelength.

Note To check the value of k, positive integer, you can also use the following calibration points of the quartz birefringence:

at $\lambda = 0.45\ \mu$m, $n_e - n_o = 0.00937$
at $\lambda = 0.70\ \mu$m, $n_e - n_o = 0.00898$
at $\lambda = 0.789\ \mu$m, $n_o = 1.5442$ and $n_e = 1.5533$.

$\rightsquigarrow$ Carefully measure all the observed (and relevant) dark fringes **for polarizers first crossed and then parallel**. Determine the values of k corresponding to each dark fringe.

Q3 Explain why the value of k is easy to determine if there are very few dark fringes (less than 2).

Q4 For all plates studied, plot the optical path difference as a function of wavelength. Deduce the value of the optical path difference at 546.1 nm (green line of mercury).

Ask the teacher to check your results.

3 Babinet compensator

A Babinet compensator is made of two birefringent prisms glued together (see figure below). The extraordinary axis of the second prism is oriented along the ordinary axis of the first prism in order to compensate its birefringence. As a result, the overall birefringence introduced by the Babinet is directly proportional to the path length difference between the two prisms. Therefore, the birefringence varies linearly with the position of the Babinet along the x axis.

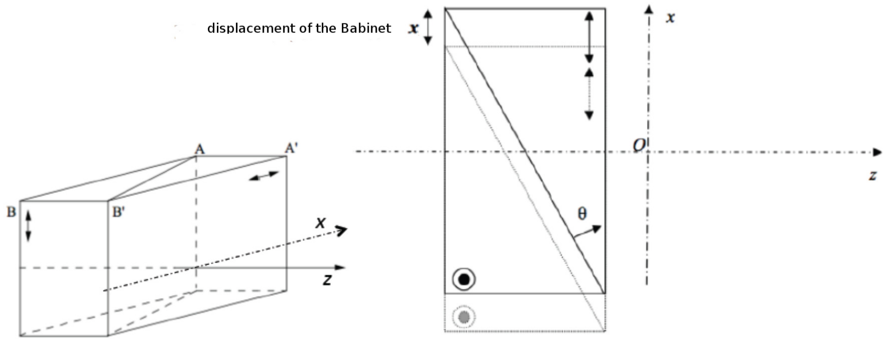


Figure 2.2: Babinet compensator

For a displacement x, the optical path difference can be written:

$$\delta_\lambda(x) = 2[n_e(\lambda) - n_o(\lambda)]\tan(\theta)x = K_{cal}(\lambda)x$$

Note that $\delta(0) = 0$.

Let us consider a Babinet compensator between crossed polarizer and analyzer. Its neutral axes are at 45° with respect to the polarizer and the analyzer axis for maximum contrast. You can then observe interference fringes equidistant and parallel to the edge of the prisms (Oy) whose interfringe in the plane xOy is equal to:

$$i(\lambda) = \frac{\lambda}{2[n_e(\lambda) - n_o(\lambda)]\tan(\theta)} = \frac{\lambda}{K_{cal}(\lambda)}$$

Under white light illumination, there are fringes following Newton's color scale with white central fringe (for $\delta = 0$) between parallel polarizers or with black central fringe between crossed polarizers.

Method for measuring birefringence: If we add between the polarizer and the analyzer a birefringent sample whose neutral axes are parallel to those of the Babinet compensator, the fringes move proportionately to the additional OPD introduced by the sample. We can then measure the shift of the Babinet compensator required to bring the central fringe back in the center of the field and deduce directly the optical path difference introduced by the sample. The transverse displacement of the Babinet compensator is measured on the vernier of the micrometer screw with high precision.

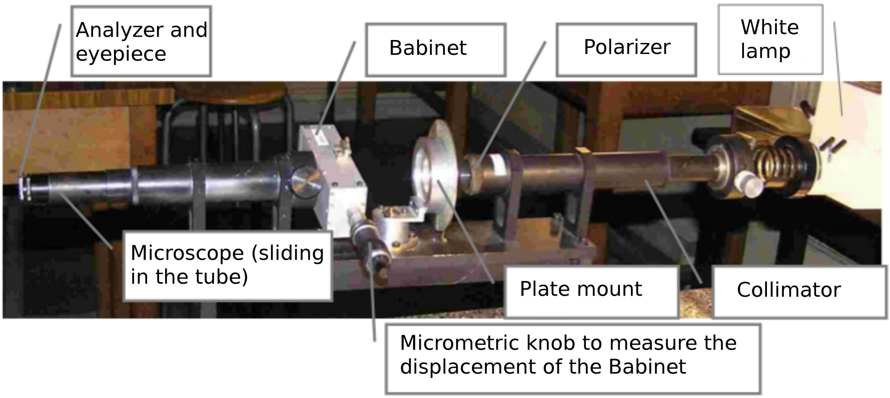


Figure 2.3: Experimental setup

3.1 Settings

⇝ Illuminate properly the Babinet compensator. Direct illumination of the slit of the collimator by the lamp, without condenser, is sufficient to cover the entire aperture of the Babinet compensator, as long as you place the lamp close enough to the slit and you orient it properly.

⇝ Without compensator, cross polarizer (located at the end of collimator) and analyzer (attached to the microscope eyepiece). Install the compensator (it slides at the entrance of the tube containing the fixed front viewfinder).

⇝ Make the focus on the crosshairs etched (the viewfinder slides inside the tube). Turn the Babinet compensator to find the extinction and then turn it of 45°. Fringes of high contrast should appear.

3.2 Calibration of the Babinet compensator

We can then **calibrate the Babinet compensator with monochromatic light** (mercury lamp equipped with a green filter). You need to measure as accurately as possible the interfringe (often called the Babinet compensator period).

Q5 Determine as precisely as possible the Babinet compensator interfringe at the wavelength of the green line of mercury. **This is about 2.4 mm. Repeat the measurement until you get close to this value.** Give the accuracy of your interfringe measurement.

This calibration allows the measurement of the optical path difference introduced by a plate at the wavelength of the green line of mercury (546.1 nm) (if this OPD is less than the maximum OPD measurable with the Babinet compensator).

Q6 What is the maximum measurable OPD with the Babinet compensator?

3.3 Measurement of the sample birefringence

⇝ Replace the mercury lamp with a white light source. The polarizer and analyzer are crossed and the direction of the axes of the compensator is at 45° with respect to the axis of the polarizers. Bring back the central dark fringe on the reticle. Press on the red button of the micrometer screw. This position will serve as a reference.

⇝ Place the plate in order to keep the dark fringe centered.

Q7 Explain why you align its neutral axes with the axis of the analyzer and polarizer.

⇝ Turn the plate by 45° around the optical axis. The black fringe is no longer centered. You must translate the compensator to bring back the dark fringe in the center. While doing so, check if the successive colors that you observe are consistent with corresponding Newton's color scale.

Q8 Measure the displacement of the Babinet compensator to bring back the dark fringe centered. Explain how this measurement allows direct calculation of the optical path difference introduced by the sample at the calibration wavelength (546.1 nm).

Note: to have a dark fringe well contrasted in the presence of the plate, it is important that the axes of the plate are well aligned with those of the Babinet compensator.

Q9 Calculate the OPD introduced by the plate at 546.1 nm. Evaluate the accuracy of this measurement.

Q10 Check that the value obtained is consistent with the values obtained by the method of channeled spectrum.

4 Measurement of the birefringence of a plate by measuring the ellipticity produced. So-called quarter-wave plate method

This method is described in the appendix 6 of lab 1.

We want to analyze the elliptic polarization produced by the studied plate by adding a QWP after it.

Q11 How should you orient the neutral axes of the QWP so that the exiting polarization state is linear? Draw a scheme with the QWP neutral axes and the linear polarization exiting it.

Experimental procedure: We use the polariscope lightened with **monochromatic light at 546.1 nm** (green line of mercury) and we cross polarizer and analyzer.
Place the studied plate with its neutral axis parallel to those of the polarizer and analyzer to find the extinction.
Above it, place, in the same way, the quarter-wave plate to keep the extinction.
Turn the plate under study by 45°. You will find the extinction again by turning the analyzer of $\beta < 90^\circ$.

⇝ Measure the angle of rotation β of the analyzer for each plate.

Q12 Show that the ellipticity is given by $\varepsilon = \beta$ or $\varepsilon = 90° - \beta$. Deduce that the phase shift introduced by the unknown plate is then given by $\varphi = 2k \times 180° \pm 2\beta$.

Q13 Calculate the optical path difference introduced by the plate at 546.1 nm, after determining k and the sign of β from a different method (channeled spectrum or Babinet compensator method).

Q14 Check that the obtained value is consistent with other measurements made.

Q15 Evaluate the accuracy of the OPD measurement.

Be mindful: If the sample is very thin (e.g. 0th order quarter-wave plate at 546.1 nm), we measure directly the optical path difference due to the sample. If it is thicker, we determine the OPD only modulo $k\lambda$. The determination of k must be made by cross-checking with other methods. This method can not usually be sufficient in itself, but allows to confirm or refine other measurements.

Present to the teacher the principle of the three measurement methods (oral presentation graded out of 5 points).

5 Conclusions on the set of measurements

Q16 For each plate, make a summary of the results obtained by the three methods. Explain why, for some samples, some methods are not appropriate.

Q17 For each plate, draw the optical path difference as a function of wavelength with the bars of uncertainty.

Q18 For each method, evaluate the accuracy of the results.

Q19 Determine the thickness of each plate, assuming that it is indeed a quartz plate cut parallel to the optical axis.
We can use the variation of $n_e - n_o$ of quartz as a function of wavelength:

$$n_e - n_o = 8.678.10^{-3} + \frac{145.025}{\lambda^2} \text{ with } \lambda \text{ in nm.}$$

6 To go further

⇝ Go back to the channeled spectrum set-up. Put the studied sample between crossed polarizer and analyzer, with its neutral axes at 45° from the polarizer axis. To make the observations easier, perform the alignment so that one neutral axis of the sample is vertical.

⇝ Incline slightly the sample so that the incident beam has a small angle with the entrance face. What is the consequence on the channeled spectrum?

⇝ Rotate the sample by 90°. The other neutral axis is vertical. Follow the same procedure and note the consequence on the channeled spectrum.

Interpret your observations.

Labwork 3

Polarimeter with a rotating analyzer. Notions of ellipsometry.

If this is your first polarization lab session, do not forget to prepare the preliminary questions page 4 (graded out of 10, they should be handed to the teacher at the beginning of the session).

Prepare question **P1** to **P3** before the session.

Contents

An ellipsometer is a widely used industrial device in particular for characterizing thin film deposition (thickness, indices). The purpose of this session is to study the working principle of a *rotating analyzer ellipsometer*. Through the first part of this session, you will become familiar with this device. **An oral presentation graded out of 5 points will end this part (section 3).** The ellipsometer is used in a second part to determine the nature of the vibration transmitted through a birefringent plate or reflected on a metal surface as a function of the incidence angle and the nature of the incident electromagnetic wave.

1 Preparation

P1 Recall the definitions of the transverse magnetic, TM and transverse electric TE polarization.

The **intensity** coefficient of reflection R_{TE} and R_{TM} versus the incidence angle are plotted on the graph on figure 3.1:

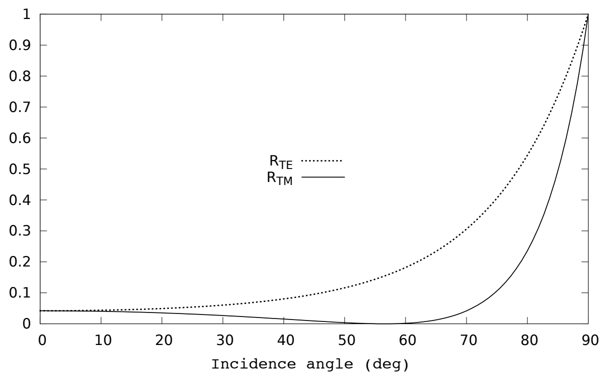


Figure 3.1: **Intensity** coefficient of reflection R_{TE} and R_{TM} versus the incidence angle (standard glass $n = 1,515$ at $633\,\text{nm}$)

P2 For what incident polarization is the intensity of the reflected beam minimal? Does this result depend on the angle of incidence?

P3 Calculate the value of Brewster's angle for a standard glass ($n = 1.515$ at 633 nm).

2 Experimental set-up

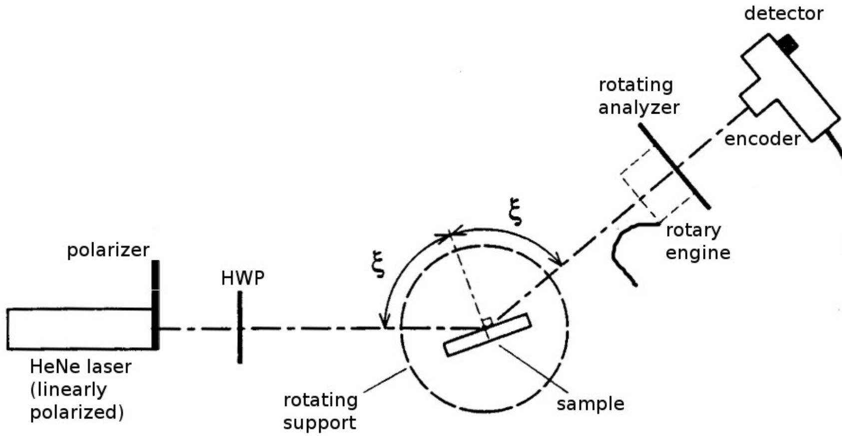


Figure 3.2: set-up scheme

The light source is a He-Ne laser linearly polarized intra-cavity. The cavity is long enough to have a stable flow (longitudinal multimode laser). To remove any residual non-polarized light and stabilize precisely the direction of polarization, a polarizer (usually oriented to maximize the flow) is placed in the laser output. *This laser is quite powerful (10 mW nominal, Class 3B), the usual precautions should be observed during the experiments.*

The detector consists of a large sensitive area photodiode ($\varnothing \simeq 11$ mm) associated with a current-voltage amplifier carefully designed to have low noise and gain as constant as possible in its useful bandwidth. It has a selector for choosing between three sensitivities. The voltage delivered by the detector is assumed to be proportional to the received flux. First, we will visualize this signal with an oscilloscope.

3 Rotating analyzer and incremental encoder of the angular position

The rotating analyzer consists of a linear polarizer, driven in rotation by an electric motor. Its rotational speed, displayed in revolutions per second on the LCD control unit of the encoder (the computer must be on), can be adjusted with the voltage of the drive motor.

An incremental encoder of angular position (orange box) is used to pinpoint the angular position of the analyzer. The model used delivers a signal, `TOP0`, which gives a single 'top' per turn, and a signal noted T, consisting of 4096 rising edges per turn. The `TOP0` signal frequency is denoted f_0 in what follows.

The incremental encoder plays an important role in the acquisition system of the signal you will use in the second part of the labwork. Details are given in appendix 3.

4 Getting started with the ellipsometer. Qualitative observations on the oscilloscope

$\rightsquigarrow$ Carefully align the arm of the goniometer supporting the rotating analyzer and the detector on the direction of the laser beam. Turn on the power supply of the detector and the computer. Send the output signal of the detector on the oscilloscope. Also send the signal `TOP0` on the second channel of the oscilloscope and trigger the sweep of the oscilloscope on this signal (very narrow 5V TTL signal).

$\rightsquigarrow$ Apply power to the drive motor of the rotating analyzer with the DC supply provided for this purpose and set the speed of rotation (viewed on the display of ENCODER MANAGEMENT BOX) to about ten turns per second. **(Avoid exceeding 25 turns/s).**

$\rightsquigarrow$ Choose the sensitivity of the detector to have a signal sufficiently strong (several volts) but not saturated on the oscilloscope. The detector delivers a voltage proportional to the normal light flux it receives apart from a very low dark voltage. (The polarizer can, if necessary, be used to limit the flux hitting the detector to avoid a saturation).

Q1 Explain the observed signals on the oscilloscope with Malus' law, especially the relationship between the frequency of the signal from the detector and the frequency f_0 of `TOP0`.

$\rightsquigarrow$ Insert a half-wave plate (HWP) at 633 nm on the beam path and make the linear polarization rotate. Observe the changes of the signal from the detector viewed on the oscilloscope.

Q2 Interpret the signal obtained as a function of the orientation of the HWP. Take particular note of the direction of travel of the signal when you rotate the HWP and explain it.

⇝ Insert now a quarter-wave plate (QWP) on the linearly polarized beam.

Q3 Interpret the signal obtained as a function of the orientation of the QWP. How can the quality of this wave plate be checked. What is the influence of a non-normal incidence of the laser beam?

Make a precise summary of your observations to the teacher (oral presentation graded out of 5 points).

5 Using the acquisition and processing software under LabVIEW

Simple measurements of phase and amplitude of a sine wave on the oscilloscope allow, strictly speaking, to determine the polarization state of a completely polarized beam hitting the rotating analyzer. However, these measurements are tedious and not very accurate. They can be very usefully computer-assisted. The objective of the next part is to get started with the acquisition software of ellipsometry measurements.

⇝ Remove the QWP before you move on.

⇝ Run the VI (Virtual Instrument) called
`TP Ellipsometry AcqEllipso.VI.`

This VI can acquire the detector signal, simultaneously view the detector signal and its FFT and calculate the parameters useful for polarimetry.

The sinusoidal signal detected can easily be written as:

$$S(r) = V_0[1 + \gamma\cos(2.2\pi r + \Phi)]$$

where r is the angular position expressed *in turns*, V_0 the average value, Φ the phase at origin and γ the modulation rate. The average value of the signal being arbitrary (it depends on the laser power and detector sensitivity), the information on the polarization is only contained in the modulation rate γ and the phase Φ. The Fourier transform of $S(r)$ whose argument is an *angular frequency* in turns^{-1} has a continuous component at *0 turn*$^{-1}$ of amplitude V_0 and a sinusoidal component at *around 2 turns*$^{-1}$ of amplitude $V_0\gamma/2$ and phase Φ:

$$S(r) = V_0 + \frac{\gamma V_0}{2}e^{i(2.2\pi r+\Phi)} + \frac{\gamma V_0}{2}e^{-i(2.2\pi r+\Phi)}$$

The two values γ and Φ are extracted in the LabVIEW program by fast Fourier transform (FFT) of the detector signal on *integer* numbers of revolutions of the analyzer. You will use the normal operation mode in the following (*orange button in the External clock position*). In that case, the acquisition of the signal $S(r)$ is synchronized with the encoder (rising edges of T). The amplitude $\gamma V_0/2$ and phase Φ values to be measured are properly calculated because the acquisition is perfectly synchronized with the sine signal at **2 turns**$^{-1}$ (cf. Appendix 3).

Note about the Fourier Transform: In fact, as a result of non-uniformities of the rotating analyzer which introduce small distortions reoccurring at every turn, the actual signal is periodic in turn, except for some fluctuations and measurement noise, and it is decomposed into Fourier series having its fundamental at ± 1 turn $^{-1}$ and harmonics at ± 2, ± 3, ... turns^{-1}. Its FFT therefore presents peaks at these particular frequencies standing out from residual background noise. The peaks at 0 and ± 2 turn^{-1} are normally highly preponderant. The quantities γ and Φ, calculated as described above may be subject to random errors related to the noise signal and systematic errors due to deterministic imperfection of the analyzer.

⇝ Without any plate on the beam path, run the VI and observe the different results displayed: acquired signal, FFT, normalized amplitude and phase of harmonic 2.

Q4 From the FFT displayed in dB (that is to say $20 \log(S(f)/S(0))$, calculate the ratio between $S(2 \text{ turns}^{-1})$ and $S(0)$, and check the value γ displayed and calculated from the acquired signal.

Q5 Calculate the ratio between the harmonic with the largest amplitude due to periodic distortions and $S(2 \text{ turns}^{-1})$. Comment.

⇝ Study the influence of stray light. What is its frequency? Its origin? Does it depend on the analyzer rotation speed?

Q6 Note the values of γ and Φ (noted Φ_{ref}) for the laser light.

⇝ Put the HWP back on the set-up.

Q7 Explain how to find precisely the direction of the neutral axes of the half-wave plate.

⇝ Carefully note down the corresponding angular graduation.

⇝ Turn the plate of $\theta = 5^\circ$ to 45° by steps of 5° and note Φ and γ. Plot $\Delta\Phi = \Phi - \Phi_{ref}$ as a function of θ.

Q8 Explain the relation between $\Delta\Phi$ and θ. Comment on the obtained results.

⇝ Put the QWP on the set-up, and orient one of its neutral axes along the incident polarization.

⇝ Turn the plate of $\theta = 5^\circ$ to 90° by steps of 5° and note Φ and γ. Plot $\Delta\Phi = \Phi - \Phi_{ref}$ and γ as a function of θ.

Q9 Note also the shape of the sine wave. Give an interpretation of the relation between $\Delta\Phi$ and θ. Comment on the obtained results and the measurement accuracy. What happens near $\theta = 45^\circ$?

The orientation of the ellipse major axis, Θ, towards a fixed reference orientation, is given by the position of the sine wave with respect to the `TOP0` pulse.

The ellipticity ε is deduced from the square root of the ratio between the minimum and the maximum of the signal.

$$\tan(\varepsilon) = b/a = \sqrt{\frac{1-\gamma}{1+\gamma}}$$

Only the handedness of the ellipse escapes the direct measurement.

Q10 Explain the two first assertions with few clear schemes. Why do we have no access to the handedness of the ellipse?

6 Getting the "absolute" direction of a linear polarization

In the previous manipulations, the "absolute" direction of the polarization is not known. The aim of this section is to determine precisely the TM polarization.

In order to optimize the determination of the TM polarization, the incidence angle has to be close to Brewster's angle for the glass we use.

The sample is a simple glass slide with very clean surface. To avoid problems of parasitic reflection on the second interface, it is blackened.

⇝ The sample will be screwed onto the support. Set, by self-collimation, the sample normal to the beam and place its surface on the axis of rotation of the stage[1].

⇝ Insert a HWP on the incident beam path. Turn then the sample with the rotation stage so that the angle of incidence is close to Brewster's angle of a standard glass.

⇝ Try to get an almost perfect extinction by varying the incident polarization with the HWP and the angle of incidence. Then, orient the arm carrying the detector so that the reflected beam hits it and refine your alignment.

You have reached the TM incident polarization. The electric field incident on the glass interface is purely horizontal. The corresponding orientation of the HWP is denoted θ_{TM}(HWP).

⇝ Write it down.

⇝ Rotate the incident linear polarization of 45° and 90°.

Q11 Why do you get still a linear polarization?

7 Introduction to ellipsometry. Study of a gold mirror

Gold Mirror: The sample must be perfectly clean to avoid distorting the measurements. Be especially mindful of fingerprints. If neces-

[1]So that the angles read on the goniometer stage holding the sample correspond to the angles of incidence, it is necessary that the surface of the sample is exactly on the vertical axis of rotation of the goniometer stage. To do this, you can rotate the sample holder on its axis and verify that the impact point of the laser beam on the sample does not move. If this is not the case, you must move forward or backward the sample holder by using the micrometer horizontal movement.

sary, ask the teacher to clean it.

All the following measurements should be taken with great care and the best possible accuracy.

⇝ Orient the HWP along θ_{TM}(HWP) again. Put this new sample on the axis of rotation of the stage.

Q12 Do you obtain then a Brewster phenomenon. Why?

⇝ *Note carefully the value* Φ_{mes} denoted hereafter Φ_{TM} and which corresponds to a horizontal incident polarization.

⇝ Now place the polarization exactly at 45° by rotating the HWP and observe first qualitatively the signal obtained.

⇝ Modify the angle of incidence.

Q13 Is the polarization state upon reflection still linear? Why?

⇝ Notice how the polarization evolves, for small (about 20°) and large (80° for example) angles of incidence.

Q14 Comment on your observations.

⇝ Note very carefully the measured values of Φ_{mes} and γ for angles of incidence between 40 and 80° (by steps of 10°) and a rectilinear incident polarization at $\Theta = 45^{\circ}$.

Introduction to ellipsometry: The previous obtained values of Φ and γ allow to mesure the gold index of refraction at the laser wavelength. The index n_{Au} of gold is complex. This means that the coefficients r_{TE} and r_{TM} are complex numbers too. One writes: $r_{TE} = |r_{TE}|\, e^{i\delta_{TE}}$ and $r_{TM} = |r_{TM}|\, e^{i\delta_{TM}}$ where δ_{TE} and δ_{TM} are the phase differences introduced on the TE and TM components of the reflected wave. Let us write $\rho = \frac{r_{TE}}{r_{TM}} = \left|\frac{r_{TE}}{r_{TM}}\right| e^{i(\delta_{TE}-\delta_{TM})} = \tan\Psi e^{i\Delta}$.

Ψ et Δ are called ellipsometric parameters. Starting from the measured values of Φ (deduced from the difference $\Phi_{mes} - \Phi_{TM}$ and γ, knowing θ, one can calculate $\tan\Psi$ and Δ (see. Appendix 1). One deduces the value of ρ^2:

$$\rho = \tan(\Psi)e^{i\Delta}$$

For bulk material (without oxide layer nor roughness), measurements of ellipsometric parameters allow to trace back to the index of the material. By a calculation given in appendix (Appendix 2), we show in particular that:

$\frac{n_1}{n_0} = \sin i_0 \sqrt{\left[1 + \left(\frac{1-\rho}{1+\rho}\right)^2 \tan^2 i_0\right]}$ where i_0 is the incidence angle, n_1 the refractive index of the material and n_0 the index of air.

The formula to calculate the ellipsometric parameters from the measured values of Φ_{mes}, Φ_{TM} and γ and for an incident polarization making an angle θ with the TM polarization, and the calculation of n_1 with these ellipsometric parameters and the angle of incidence i_0 were implemented in a Matlab code.

⇝ With the Matlab code and your experimental results, calculate the optical index of gold at the laser wavelength.

Q15 Compare it with the value published in the "Handbook of Optical Constant of Solids", edited by E. D. Palik: $n_1 = 0.2 + i3.3$.

To illustrate, the following figure shows a calculation of the modulation rate and the ellipsometric parameter Δ as a function of the incidence angle (for an incident polarization at 45°). This calculation is made from the value of $n_1 = 0.2 + i3.3$ of the index at 633 nm. The vertical dotted line represents the transition to $\Delta = \pi/2$: knowing that $\tan\psi \simeq 1$, *the polarization is almost circular.*

[2] We do not have access to the sign of Δ which depends on the handedness of the ellipse and may be determined by further measurements.

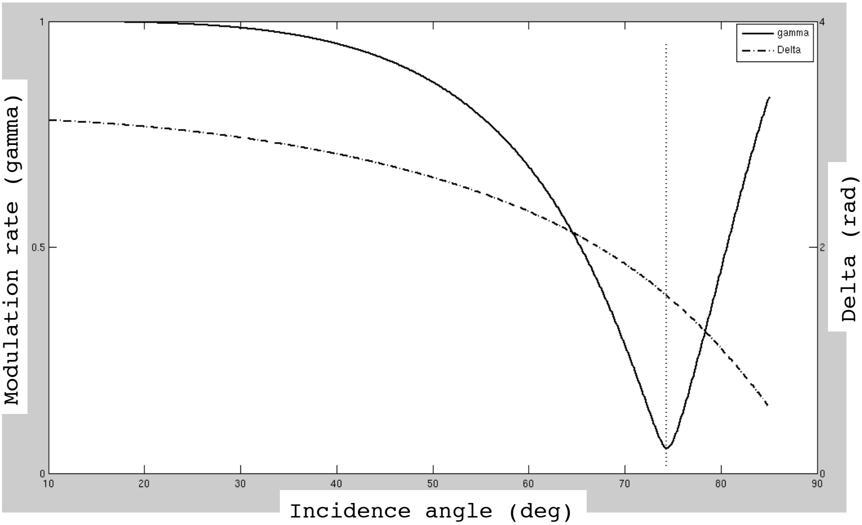


Figure 3.3: Graph of the modulation rate and the ellipsometric parameter Δ as a function of the incidence angle

8 Appendix 1: ellipsometric angles

8.1 Definition of the ellipsometric angles Ψ and Δ

Consider a stratified planar system, that is to say composed of parallel layers of media having different indices. When this system is illuminated under oblique incidence, it is possible that the polarization of the reflected wave is modified with respect to the polarization of the incident wave. This modification comes from the difference between Fresnel's amplitude coefficient of reflectivity r_{TE} for the Transverse Electric polarization (**E** perpendicular to the incidence plane) and r_{TM} for the Transverse Magnetic polarization (**E** contained in the incidence plane)[3]. These two linear polarizations are the *eigen* polarization states of the system if all the layers of the stratified structure are isotropic and show no rotatory activity (cf.

[3]One can show that the difference between the two reflectivity coefficients reaches a maximum for incidence angles close to Brewster's angle for the substrate.

electromagnetism class 1A). The parameters ρ, Ψ and Δ are defined as follows:

$$\rho = \frac{r_{TE}}{r_{TM}} = \tan(\Psi)\exp(i\Delta) \text{ with } \Psi \in [0°, 90°] \text{ and } \Delta \in]-180°, +180°]$$

The angles Ψ and Δ are called the ellipsometric angles. They have a straightforward meaning:

- $\tan\Psi$ characterizes the **attenuation ratio** (of complex amplitudes, $\tan^2\Psi$ of "intensities") between a TE and a TM polarized wave when reflected on the sample.
- Δ characterizes the relative phase shift of a TE polarized wave with respect to a TM one when reflected on the sample.

The so-called ellipsometry consists firstly in determining precisely by the experiment the angles Ψ and Δ. The measurements are usually performed for several incidence angles and different wavelengths. The second step allows to deduce from these raw data the characteristics of the sample layers.

8.2 Experimental determination of the ellipsometric angles

The ellipsometric angles can be determined from any polarimeter. This part develops the calculations in the case of a polarimeter with a rotating analyzer.

Definition of the axes x, y and z The z axis is defined as the direction of propagation of the incident wave. The x axis is such that the xz plane is the incidence plane. The y axis is perpendicular to the incidence plane. In that frame, the electric field of a TM polarized wave is along the x axis while the electric field of a TE polarized wave is along the y axis (see Figure 1). After reflection, the axes are called x', y' and z'.

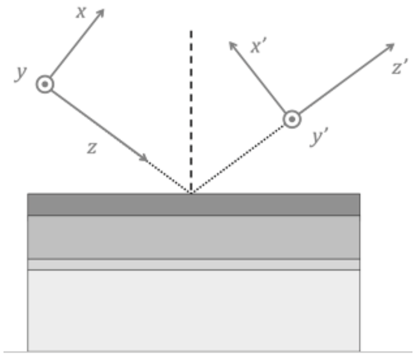


Fig. 1. Stratified system under study. The electric field of the incident wave is in the xy plane. If the electric field is along the x axis, the wave is TM polarized ; if it is along the y axis, the wave is TE polarized.

Calculation of the light flux exiting the rotating analyzer Let us consider an incident wave, linearly polarized along a direction having an angle θ with respect to the x axis. The complex amplitude of the electric field vector of the incident wave is proportional to:

$$E_x = E_0 \cos\theta$$
$$E_y = E_0 \sin\theta$$

The complex amplitude of the electric field vector of the wave reflected on the stratified sample is thus written:

$$\begin{pmatrix} E_{x'} \\ E_{y'} \end{pmatrix} = \begin{pmatrix} r_{TM} E_x \\ r_{TE} E_y \end{pmatrix} = \begin{pmatrix} r_{TM} E_0 \cos\theta \\ r_{TE} E_0 \sin\theta \end{pmatrix} = r_{TM} E_0 \begin{pmatrix} \cos\theta \\ \tan\Psi \exp(\mathrm{i}\Delta) \sin\theta \end{pmatrix}$$

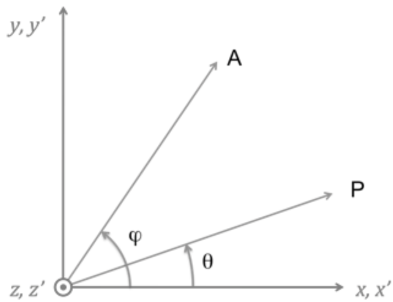


Fig. 2. How to write the orientations of the polarization states before and after reflection. z and z' are directed toward the reader (convention of the "light coming to you"). P corresponds to the direction of the linear polarization of the incident beam. A corresponds to the (instantaneous) direction of the analyzer.

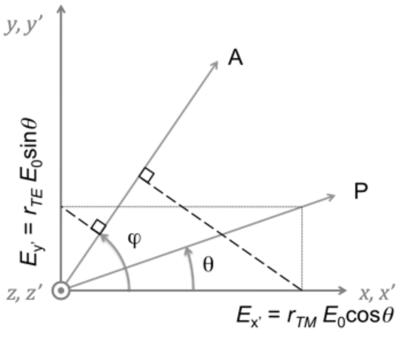


Fig. 3. Projection of the reflected fields on the direction of the analyzer.

The complex amplitude, written a, of the wave exiting the analyzer A is obtained by projecting the components $E_{x'}$ and $E_{y'}$ of the reflected field on the direction of the analyzer (figure 3) :

$$a = E_{x'} \cos\varphi + E_{y'} \sin\varphi \;\propto\; \cos\theta \cos\varphi + \tan\Psi \exp(\mathrm{i}\Delta) \sin\theta \sin\varphi$$

The flux received by the detector is proportionnal to the squared module of the complex amplitude a of the wave exiting the analyzer. Let us write a^*

the complex conjugate of a :

$$\begin{aligned} F &\propto aa^* \\ &\propto [\cos\theta\cos\varphi + \tan\Psi\exp(\mathrm{i}\Delta)\sin\theta\sin\varphi][\cos\theta\cos\varphi + \tan\Psi\exp(-\mathrm{i}\Delta)\sin\theta\sin\varphi] \end{aligned}$$

The development and the simplification of the previous equation gives:

$$F(\varphi) \propto \tan^2\Psi\sin^2\theta + \cos^2\theta + \cos(2\varphi)[\cos^2\theta - \tan^2\Psi\sin^2\theta] + \sin(2\varphi)\sin(2\theta)\tan\Psi\cos\Delta$$

This expression shows that the flux received by the detector is a **sine wave** of the azimuth φ of the analyzer, **with a period of** 180°. If the analyzer rotates evenly, the detected flux varies thus in time as a sine wave whose frequency is twice the one of the analyzer rotation speed, noted f.

The division of the previous equation by the part independent of φ gives:

$$F(\varphi) \propto 1 + \alpha\cos(2\varphi) + \beta\sin(2\varphi)$$

with

$$\alpha = \frac{\cos^2\theta - \tan^2\Psi\sin^2\theta}{\tan^2\Psi\sin^2\theta + \cos^2\theta} = \frac{1 - \tan^2\Psi\tan^2\theta}{1 + \tan^2\Psi\tan^2\theta}$$

and

$$\beta = \frac{sin(2\theta)\tan\Psi\cos\Delta}{\tan^2\Psi\sin^2\theta + \cos^2\theta} = \frac{2\tan\theta\tan\Psi\cos\Delta}{1 + \tan^2\Psi\tan^2\theta}$$

α and β are the Fourier coefficients of the sine wave at the frequency $2f$. One can also write:

$$F(\varphi) \propto 1 + \alpha\cos(2\varphi) + \beta\sin(2\varphi) = 1 + \gamma\cos(2\varphi + \phi)$$

with $\alpha = \gamma\cos\phi$ and $\beta = -\gamma\sin\phi$.

The values γ and ϕ, and thus α and β can be very easily deduced from the measurements as displayed by the VI, as long as you have understood that φ has the direction x (x') for origin, that is to say the direction of the TM polarization in the previous calculations (cf. Fig.3) and not the arbitrary one given by `TOP0`.

Practical calculation of Ψ and Δ The parameter ϕ defined above can be easily determined experimentally from the reference phase ϕ_{TM} for the TM polarization TM and from the phase ϕ_θ given by the reflected beam when the incident beam is linearly polarized along a direction making an angle θ with the direction x' (TM), and from the direction of rotation of the analyzer using $\phi = \pm(\phi_\theta - \phi_{TM})$ (+ if the analyzer rotated anti clockwise, - if not).

The coefficients α and β are then calculated using the measurement of γ of the reflected beam when the incident beam is linearly polarized along a direction making an angle θ with the direction x (we still have $\alpha = \gamma\cos\phi$ and $\beta = -\gamma\sin\phi$). One can now find the value of ρ using the fact that:

$$\tan\Psi = \sqrt{\frac{1-\alpha}{1+\alpha}}\frac{1}{|\tan\theta|} \text{ et } \cos\Delta = \beta\frac{1+\tan^2\Psi\tan^2\theta}{2\tan\theta\tan\Psi}$$

Δ's sign, connected to the handedness of the elliptic polarization, cannot be determined by the previous experiments. To disambiguate, we must introduce an additional measurement with a wave plate of slow and fast axes known, usually called *compensator*.

9 Appendix 2: Calculation of the index of a metallic mirror

The equations giving Fresnel's reflection coefficients in the TE/TM system for a planar interface between two semi-infinite media denoted by 0 and 1 are :

$$r_{TE} = \frac{\gamma_0 - \gamma_1}{\gamma_0 + \gamma_1} \text{ and } r_{TM} = \frac{\varepsilon_1\gamma_0 - \varepsilon_0\gamma_1}{\varepsilon_1\gamma_0 + \varepsilon_0\gamma_1}$$

with $\varepsilon_i = n_i^2$. We introduce ρ, being the ratio r_{TE}/r_{TM}.

$$\rho = \frac{\varepsilon_1\gamma_0 + \varepsilon_0\gamma_1}{\varepsilon_1\gamma_0 - \varepsilon_0\gamma_1}\frac{\gamma_0 - \gamma_1}{\gamma_0 + \gamma_1}$$

One can easily show that:

$$\left(\frac{1-\rho}{1+\rho}\right)^2 = \frac{\gamma_0^2\gamma_1^2(\varepsilon_1 - \varepsilon_0)^2}{(\varepsilon_1\gamma_0^2 - \epsilon_0\gamma_1^2)^2} = \frac{1}{\tan^2 i_0}\left(\frac{\varepsilon_1}{\varepsilon_0\sin^2 i_0} - 1\right)$$

From the last equation, one gets the expression of the ratio $\varepsilon_1/\varepsilon_0$:

$$\frac{\varepsilon_1}{\varepsilon_0} = \sin^2 i_0\left[1 + \left(\frac{1-\rho}{1+\rho}\right)^2\tan^2 i_0\right]$$

10 Appendix 3: Acquisition software and FFT

By using the external clock synchronization, the acquisition of the signal $S(r)$ is synchronized with the encoder (rising edges of T). The sampling period is $P_s = 1/4096$ turn, corresponding to a sampling rate $f_s = 4096$ turns^{-1}. The number of points depends on the number of turns N_t during which the detector signal is acquired:

$$N_s = N_t \times 4096 = N_t \times f_s$$

The FFT calculation requires that the number of points calculated on the spectrum between 0 and f_s should be identical to the number of points of the sampled signal, that is to say N_s. The FFT is thus calculated with a step of: $\frac{f_s}{N_s} = \frac{f_s}{N_t \times f_s} = \frac{1}{N_t}$. The signal of frequency 2 turns^{-1} of interest here is therefore **exactly** the point $\mathbf{2N_t}$. For example, if the number of turns is 4, this is the eighth point of the FFT. The value of amplitude $\gamma V_0/2$ and the value of phase Φ to be measured are correctly calculated thanks to this perfect synchronization of the acquisition with the sinusoidal signal of frequency 2 *turns*$^{-1}$.

Another way to present the same result is to note that the acquisition and sampling window of the signal corresponds exactly to an integer number of revolutions of the analyzer.

Labwork 4

Study of an electro-optic modulator

If this is your first polarization lab session, do not forget to prepare the preliminary questions page 4 (graded out of 10, they should be handed to the teacher at the beginning of the session).

The aim of this session is to study the working principle of an electro-optic modulator and the way it is used.

No written report is asked for this session. You will just have to fill out a result sheet. The questions to be answered on the sheet are marked with a $\diamond$. The questions **P1** to**P4**must be prepared before the session.

Contents

1 Preparation: the electro-optic effect

The electro-optic effect will be seen during the second semester in lecture and in tutorials. The few items below allow us to carry out the labwork on and to understand the birefringence induced by an electric field and the use of an electro-optical component as an intensity modulator.

The application of an electric field on a non-centrosymmetric crystal can cause a change in the refractive index. If the change of index is proportional to the applied field, this phenomenon is called the Pockels effect (this is the case of the KD*P crystal studied in this lab). If, however, the change is proportional to the square of the applied field, this is called the Kerr effect.

The electro-optical effect is thus an effect of electrically induced birefringence. The crystal behaves as a birefringent plate with a slow axis and a fast axis whose indices vary depending on the applied voltage. We describe these variations by changing the index ellipsoid.

In an arbitrary coordinate system $Oxyz$ the equation of the index ellipsoid is:

$$\frac{x^2}{n_{xx}^2}+\frac{y^2}{n_{yy}^2}+\frac{z^2}{n_{zz}^2}+\frac{2xy}{n_{xy}^2}+\frac{2xz}{n_{xz}^2}+\frac{2yz}{n_{yz}^2}=1$$

In the coordinate system $OXYZ$ of the medium neutral axis, one obtains:

$$\frac{X^2}{n_{XX}^2}+\frac{Y^2}{n_{YY}^2}+\frac{Z^2}{n_{ZZ}^2}=1$$

The electro-optical effect results in a slight variation of the indices: the coefficients $1/n_{ij}^2$ undergo variations $\Delta(1/n_{ij}^2)$ and become the coefficients $1/n_{ij}'^2$:

$$\frac{1}{n_{ij}'^2}=\frac{1}{n_{ij\,(E=0)}^2}+\Delta\left|\frac{1}{n_{ij}^2}\right|$$

The variations of the coefficients $1/n_{ij}^2$ are calculated by taking the product of the (6 x 3) matrix of the electro-optical coefficients r_{ij}, which depend on the nature of the crystal, by the electric field vector, **E**. For KD*P, which, without any applied electric field is a uniaxial crystal along Oz, the initial ellipsoid has the following equation:

$$\frac{x^2}{n_o^2}+\frac{y^2}{n_o^2}+\frac{z^2}{n_e^2}=1 \text{ with } n_o=1.51 \text{ and } n_e=1.47 \text{ at } \lambda=0.6\mu\text{m}.$$

The symmetry properties of the KD*P crystal allow to show that the matrix of the electro-optic coefficients is:

$$\begin{pmatrix}\Delta\left[\frac{1}{n_{xx}^2}\right]\\ \Delta\left[\frac{1}{n_{yy}^2}\right]\\ \Delta\left[\frac{1}{n_{zz}^2}\right]\\ \Delta\left[\frac{1}{n_{yz}^2}\right]\\ \Delta\left[\frac{1}{n_{zx}^2}\right]\\ \Delta\left[\frac{1}{n_{xy}^2}\right]\end{pmatrix}=\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\\r_{41}&0&0\\0&r_{41}&0\\0&0&r_{63}\end{pmatrix}\begin{pmatrix}E_x\\E_y\\E_Z\end{pmatrix}$$

with $r_{41}=8.8.10^{-12}\text{m.V}^{-1}$ and $r_{63}=26.2.10^{-12}\text{m.V}^{-1}$

It can be easily shown that the ellipsoid of KD*P in the presence of an electric field E_z applied along Oz becomes:

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2r_{63}E_z xy = 1,$$

where the appearance of a crossed term indicates a rotation of the ellipsoid. In the presence of an electric field along Oz, the neutral axes Ox' and Oy' are at 45° to the axis Ox and Oy.

Making a change of variables one can obtain the equation of the ellipsoid in these new neutral axes:

$$\begin{cases} x' &= \frac{1}{\sqrt{2}}(x+y) \\ y' &= \frac{1}{\sqrt{2}}(y-x) \end{cases} \Rightarrow \left(\frac{1}{n_o^2} + r_{63}E_z\right) x'^2 + \left(\frac{1}{n_o^2} - r_{63}E_z\right) y'^2 + \frac{z^2}{n_e^2} = 1$$

At first-order the equation of the ellipsoid in these new neutral axes can be written as

$$\frac{x'^2}{n_{x'}^2} + \frac{y'^2}{n_{y'}^2} + \frac{z^2}{n_e^2} = 1 \text{ with } \begin{cases} n_{x'} &= n_o - \frac{1}{2} n_o^3 r_{63} E_z \\ n_{y'} &= n_o + \frac{1}{2} n_o^3 r_{63} E_z \end{cases}$$

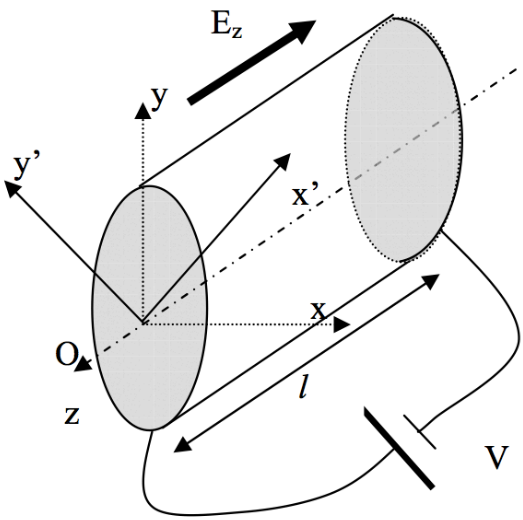


Figure 4.1: Crystal sheme.

In the presence of a field E_z, we get thus two neutral axes Ox' and Oy' in the plane perpendicular to the z-axis, characterized by a birefringence:

$$\Delta n = n_{y'} - n_{x'} = n_o^3 r_{63} E_z$$

Let us consider a monochromatic plane wave linear along Oy propagating in the crystal along the direction Oz.

P1 ◇ Give the expression of the phase shift introduced and show that it is independent of the length l of the crystal. Give the expression of the voltage V_π for which the crystal behaves like a half-wave plate.

P2 Suggest a set-up using the KD*P crystal for modulating the amplitude of a linearly polarized electromagnetic wave.

P3 ◇ Suggest a set-up using the KD*P crystal for modulating the phase of a linearly polarized electromagnetic wave.

P4 ◇ Look for application examples of electro-optic modulators on the internet.

2 Caracterization of the electro-optic modulator

Align the following set-up:

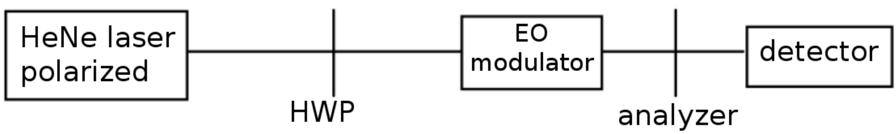


Figure 4.2: Experimental setup

2.1 Setting the optical axis of the modulator with respect to the laser beam

⇝ Cross the analyzer with the linear polarization state produced by the half-wave plate (HWP).

$\rightsquigarrow$ Install and align the electro-optic modulator on the laser beam (for the moment, the modulator is not powered).

$\rightsquigarrow$ Observe the interference pattern obtained after the analyzer.

$\rightsquigarrow$ Adjust the orientation of the modulator in order to align the center of the black cross and the spot of the laser beam transmitted by the analyzer.

2.2 Setting when the modulator is powered

The high voltage power supply (HV) provides a voltage between 0 and 3000 V.

Warning High Voltage

- Never disconnect a cable when the power is on.
- Use only high-voltage coaxial cables (green) whose core is well protected.
- Never connect a high voltage cable directly to a low voltage cable.

The HV power supply can be adjusted with a potentiometer.

$\rightsquigarrow$ Apply to the crystal a DC high voltage close to 1500 V.

$\rightsquigarrow$ Orient the neutral axes of the crystal, Ox' and Oy', at 45° with respect to the analyzer axis.

For this, two methods are possible:

1. Search the extinction by rotating the modulator around its axis. Then, starting from the extinction, turn the modulator by 45° around its axis.
2. Do not touch the modulator (in order not to misalign it). Search the extinction by rotating the half-wave plate and the analyzer. Then turn off the HV power supply and turn the analyzer by 45°, then rotate the half-wave plate to recover the extinction.

Q1 Explain and comment on the method of alignment you have chosen.

⇝ Visually check the intensity variation obtained by varying the voltage applied to the KD*P crystal.

2.3 Study of the characteristic of the transmitted flux as a function of the applied voltage

⇝ ◇ Measure the evolution of the light intensity exiting the analyzer with the HV (between 0 and 3000 V) applied to the modulator.

⇝ ◇ Evaluate the modulation rate obtained, defined by:

$$\eta = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$$

⇝ ◇ Measure the high voltage corresponding to the maximum transmission. For this voltage, precise the polarization state produced by the modulator. Explain why this voltage is called V_π.

Q2 ◇ Deduce for the measured value of V_π the value of r_{63}, taking into account the fact that the studied modulator is made of two identical crystals in series, subjected to the same field E_z, and whose phase differences add themselves.

2.4 Study of the polarization state produced by the crystal

Q3 ◇ For what value of the applied voltage is the electrooptic crystal equivalent to a half-wave plate? to a quarter-wave plate?

⇝ ◇ Check the polarization state at the exit of the crystal for these two voltage values, by explaining the method.

⇝ ◇ Apply a voltage of 700 V, and then of 1800 V to the crystal. For each case, determine the polarization state obtained. Find out the position of the major and minor axes of the ellipse and measure its ellipticity by a photometric measurement.

Q4 ◇ Deduce the phase difference introduced by the crystal and check that the measured phase difference is consistent with the expected values.

Present the obtained results to the teacher (oral presentation graded out of 5 points).

2.5 Use of the electro-optic crystal as a linear modulator of flux

Q5 ◇ Around what operating point of the characteristic previously obtained can the electro-optic crystal be used as a linear modulator of flux?

We will replace the power supply, that can not be modulated, by a low frequency generator. *In practice: a small blue box for low to high voltage adaptation is used to send the voltage delivered by the low frequency generator directly to the modulator.*
To be in the linear region of the characteristic, you can add a wave plate just before the modulator.

Q6 ◇ What kind of wave plate has the same effect as the previous high voltage applied to the modulator? How should one orient this plate to stay around the operating point chosen in the previous section?

Q7 Explain how to perform this setting.

Q8 ◇ What is the order of magnitude of the set-up bandwidth? Use the oscilloscope to perform this measurement.

⇝ ◇ Use this setup to send through the laser beam a modulation in the audio bandwidth from the tape player mini system.

Q9 Comment and interpret. Show your set-up to the teacher for validation.

⇝ ◇ Measure the modulation rate obtained for a voltage of 20 V peak to peak applied to the crystal. Check that this modulation rate is consistent with the characteristic obtained previously.

Q10 ◇ Comment on the evolution of the flux modulation when changing the orientation of the plate.

If you still have time and curiosity

Remove the plate. Observe, discuss and analyze the intensity distribution obtained at the exit of the analyzer with a non-powered modulator for an incident polarization and the analyzer crossed (black cross observed at the beginning of the session).