Computational Optical Imaging - Optique Numerique

-- Volumetric 3D –

Winter 2013

Ivo Ihrke

Overview

- Tomography
  - Absorption / emission
  - Fourier Slice Theorem and Filtered Back Projection
  - Algebraic Reconstruction
  - Applications
- Volume Slicing
  - Direct Scanning
  - Index Matching
  - Bessel Beams

Volumetric 3D

Tomography

Outline

- Computed Tomography (CT)
  - Radon transform
  - Filtered Back-Projection
  - natural phenomena
  - glass objects

Computed Tomography (CT)

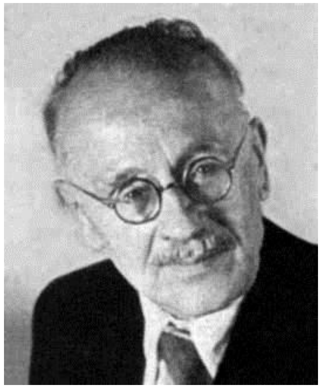
3D

Some History

- Radon transform (1917)

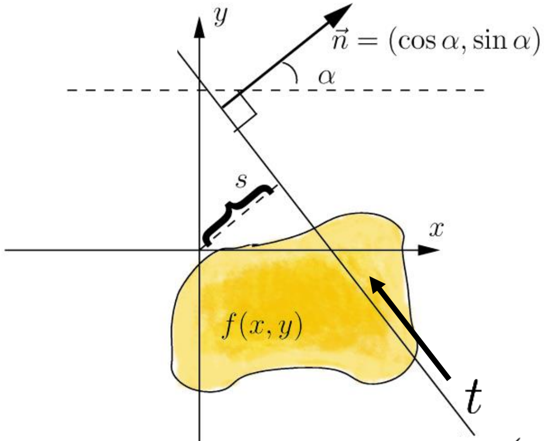
$$\mathcal{R}\{f\}(\alpha, s) = \int_{c_{\alpha,s}} f \circ c_{\alpha,s}(t)dt$$

  - Radon: Inverse transform exists if all (α, s) are covered
  - First numerical application Viktor Ambartsumian (1936, astrophysics)



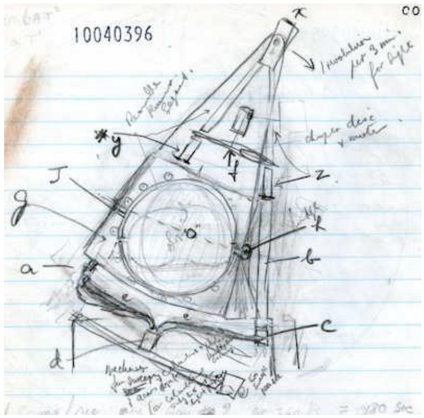
Johann Radon (1887-1956)

Viktor Ambartsumian (1909-1996)



Some History

- CT Scanning

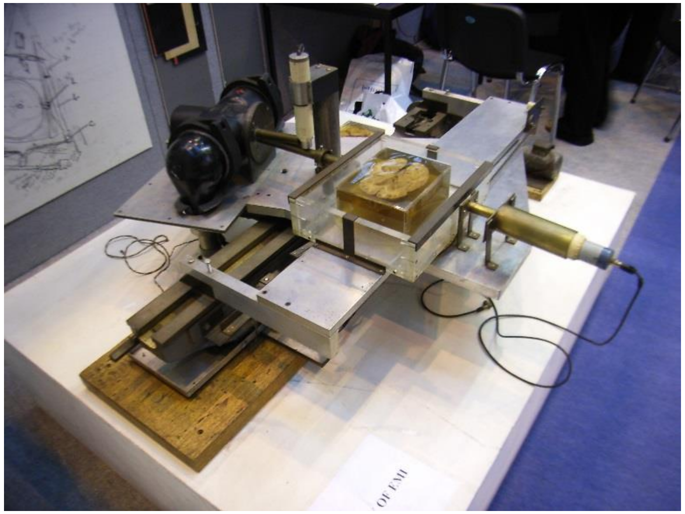


Sketch of the invention

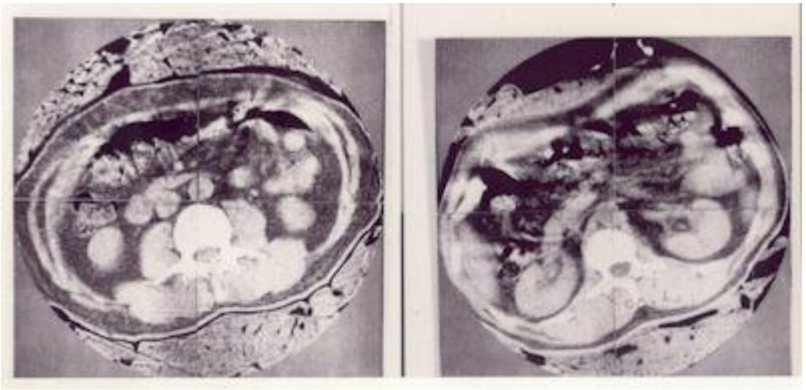
Godfrey Hounsfield (1919-2004)

Allan Cormack (1924-1998)

- 1979 Nobel prize in Physiology or Medicine



Prototype scanner

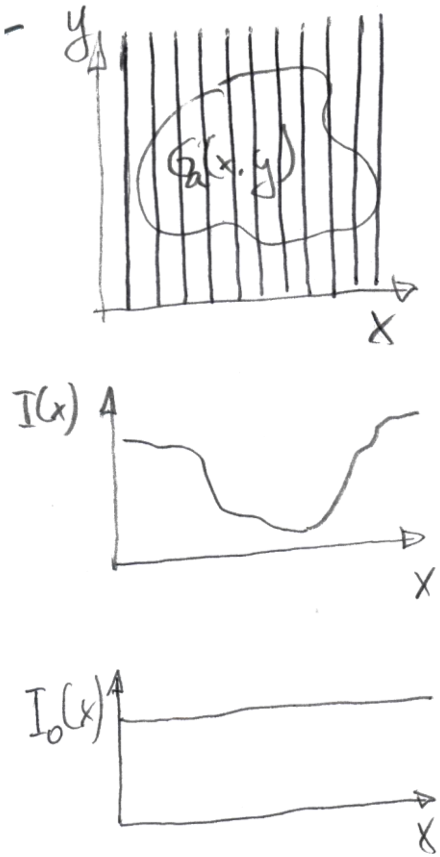


Hounsfield’s abdomen

The math

- X-rays are attenuated by body tissue and bones
  - Attenuation is spatially variant (attenuation coeff. $\sigma_a(x,y)$)

$$
\begin{aligned}
I(x) &= I_0(x) e^{-\int_c \sigma_a(x,y)dy} \\
\Rightarrow \quad \frac{I(x)}{I_0(x)} &= e^{-\int_c \sigma_a(x,y)dy} \\
\Rightarrow \quad \log \frac{I(x)}{I_0(x)} &= -\int_c \sigma_a(x,y)dy
\end{aligned}
$$



  - $I(x), I_0(x)$ are known, determine $\sigma_a(x,y)$
  - Ill-posed for only one direction α
    - Need all

Well-Posed and Ill-Posed Problems

- Definition [Hadamard1902]
  - a problem is well-posed if
    1. a solution exists
    2. the solution is unique
    3. the solution continually depends on the data
  - a problem is ill-posed if it is not well-posed

Volumetric 3D

Tomography – Fourier-Based Techniques

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Computed Tomography

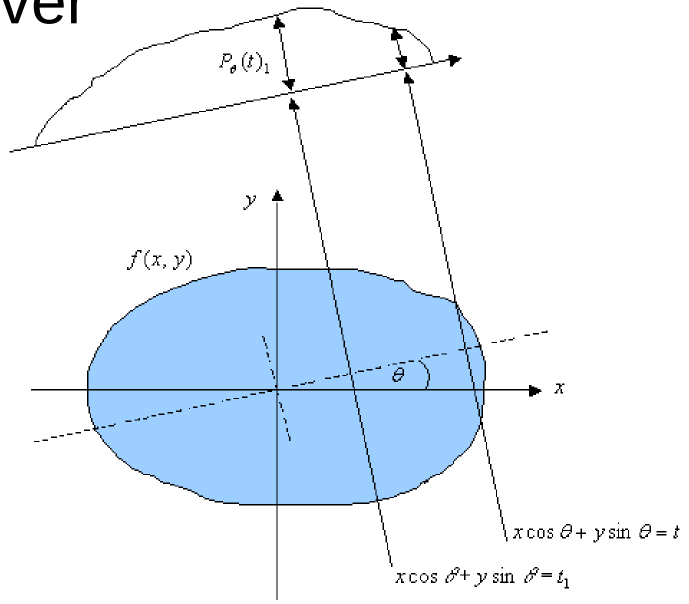
- tomography is the problem of computing a function from its projections
- a projection is a set of line integrals over function m along some ray c

$$o = \int_c m(c(s))ds$$

- invert this equation (noise is present)

$$o = \int_c m(c(s))ds + n$$

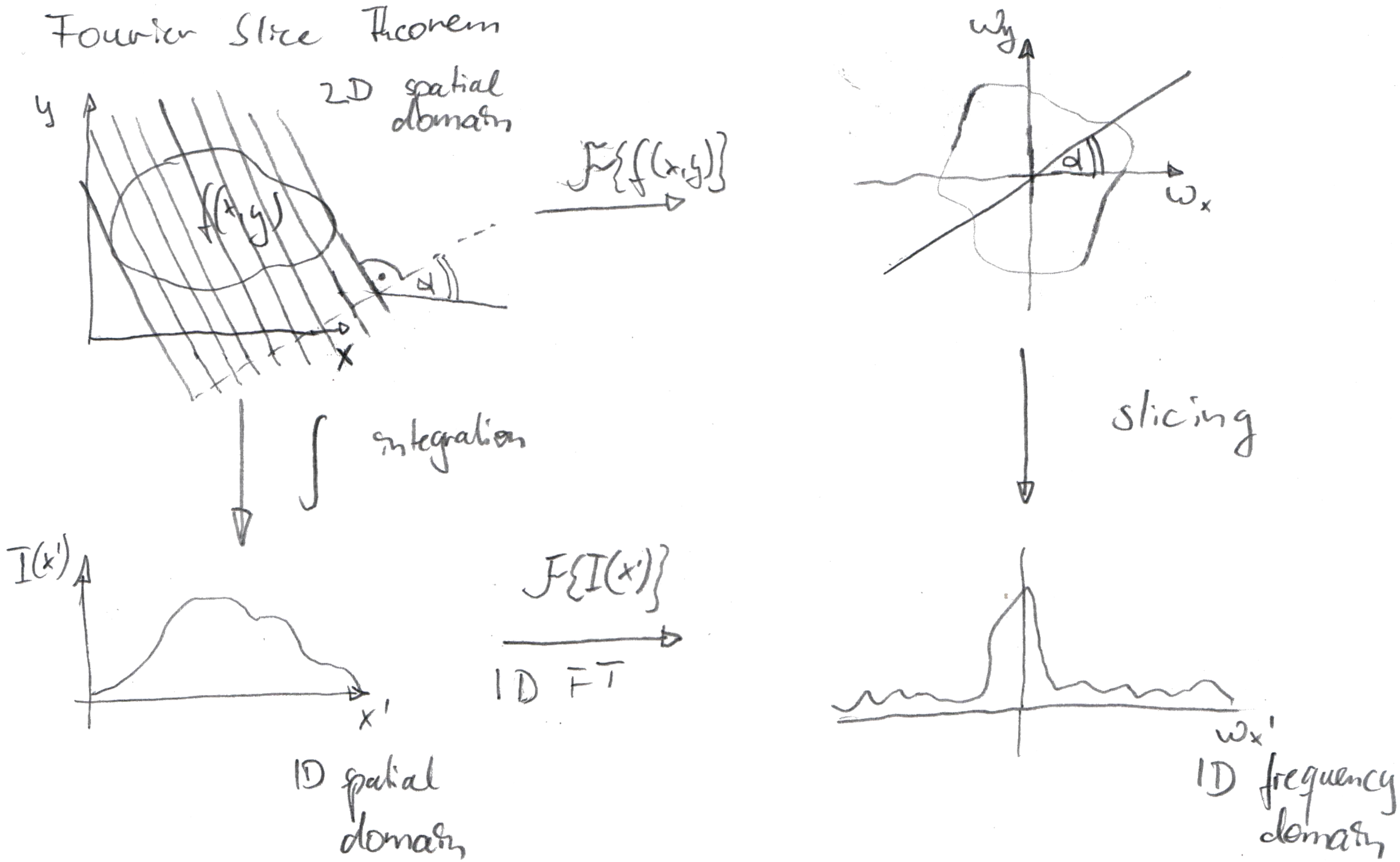
- if infinitely many projections are available this is possible (Radon transform) [Radon1917]



Computed Tomography – Frequency Space Approach

- Fourier Slice Theorem
- The Fourier transform of an orthogonal projection is a slice of the Fourier transform of the function

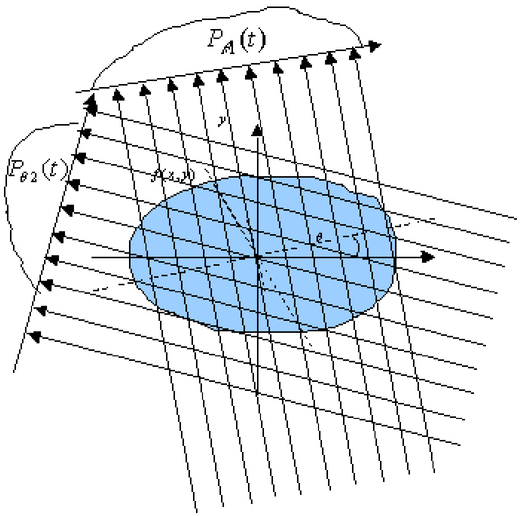
Computed Tomography – FST



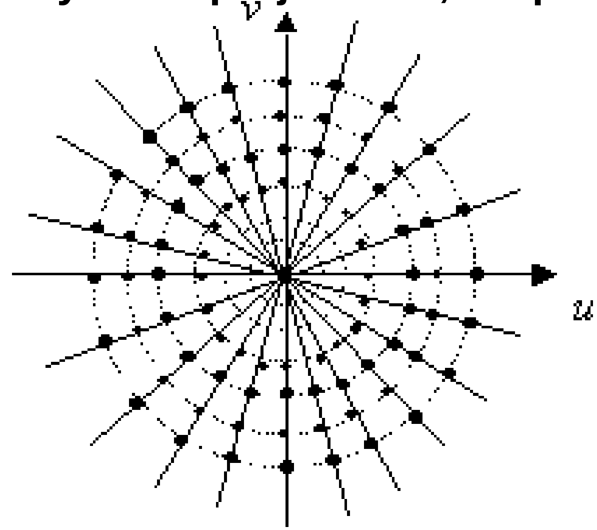
Computed Tomography – Frequency Space Approach

- for recovery of the 2D function we need several slices

several projections, spatial domain



many more projections, frequency domain



- slices are usually interpolated onto a rectangular grid
- inverse Fourier transform
- gaps for high frequency components

  → artifacts

Frequency Space Approach - Example

without noise !

original (Shepp-Logan head phantom)

reconstruction from 18 directions

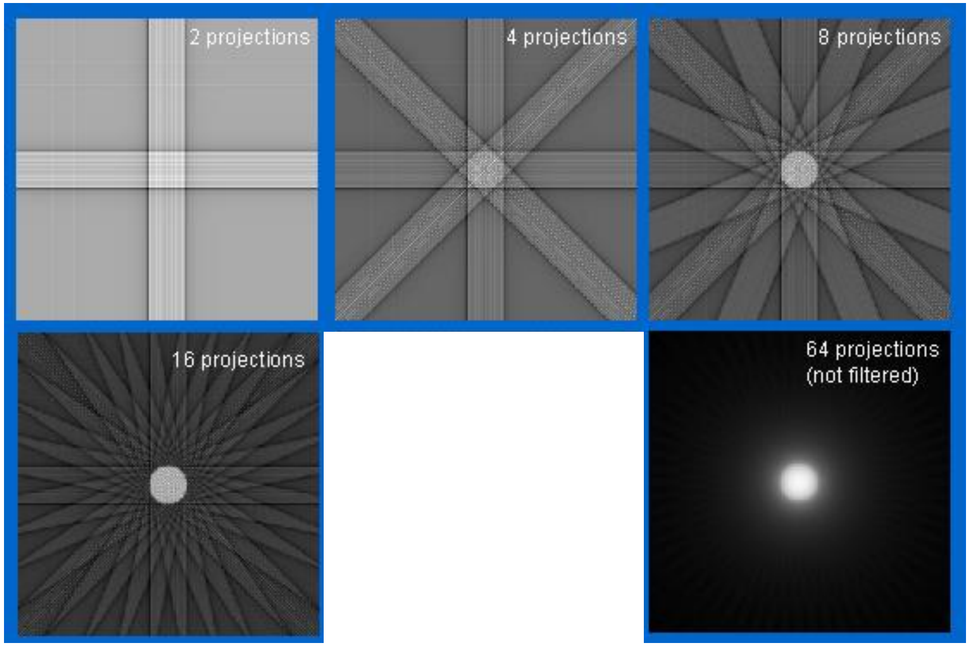
reconstruction from 36 directions

reconstruction from 90 directions

Filtered Back-Projection

- Fourier transform is linear
  - → we can sum the inverse transforms of the lines in frequency space instead of performing the inverse transform of the sum of the lines

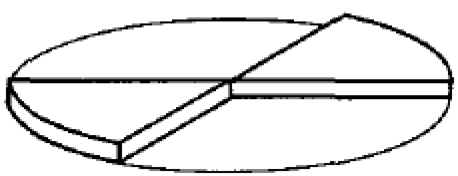
backprojection:



Filtered Back-Projection

- Why filtering ?
- discrete nature of measurements gives unequal weights to samples
- compensate

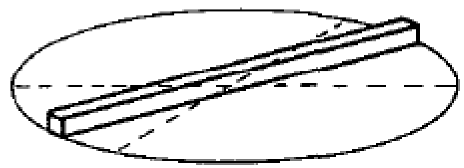
would like to have wedge shape for one discrete measurement



frequency domain

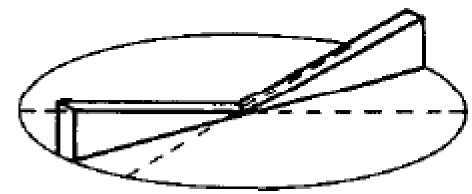
(a)

have a bar shape (discrete measurement)



(b)

compensate to have equal volume under filter

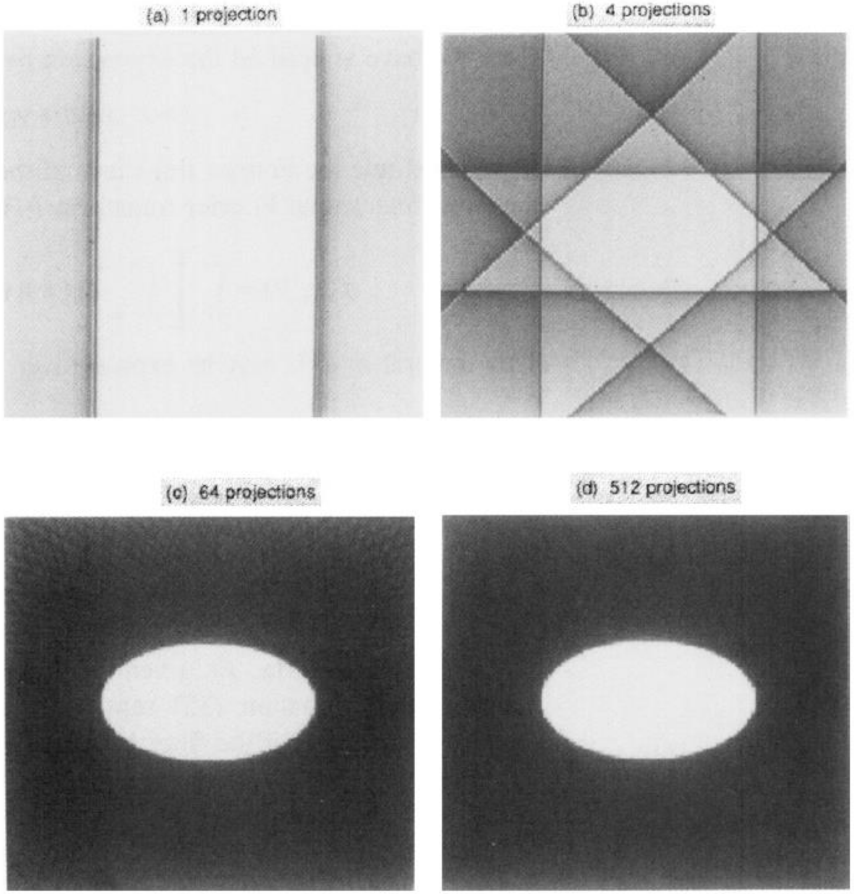


high pass filter

(c)

Filtered Back-Projection (FBP)

- high pass filter 1D projections in spatial domain
- back-project
- blurring is removed
  - FBP can be implemented on the GPU
  - projective texture mapping



Frequency Space based Methods

- Advantages
  - Fast processing
  - Incremental processing (FBP)
- Disadvantages
  - need orthogonal projections
  - sensitive to noise because of high pass filtering
  - Frequency-space artifacts, e.g. ringing
  - Equal angular view spacing (or adaptive filtering)

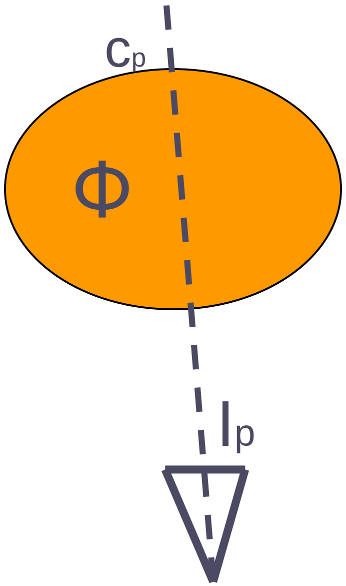
Volumetric 3D

Tomography – Algebraic Techniques

Algebraic Reconstruction Techniques (ART)

- object described by Φ, a density field of e.g. emissive soot particles
- pixel intensities are line integrals along line of sight

$$I_p = \int_c \phi \, ds$$

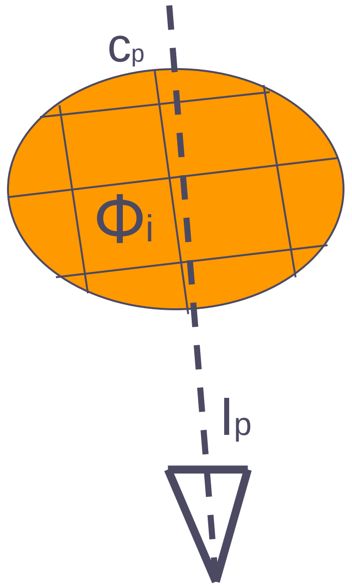


- Task: Given intensities, compute Φ

ART

- Algebraic Reconstruction Technique (ART)
- Discretize unknown Φ using a linear combination of basis functions Φ_i

$$I_p = \int_c \left(\sum_i a_i \phi_i \right) ds$$



- → linear system p = Sa

$$I_p = \sum_i a_i \left(\int_{c_p} \phi_i \, ds \right)$$

ART

- Discretize unknown Φ using a linear combination of basis functions Φ_i

$$I_p = \int_c \left(\sum_i a_i \phi_i \right) ds$$

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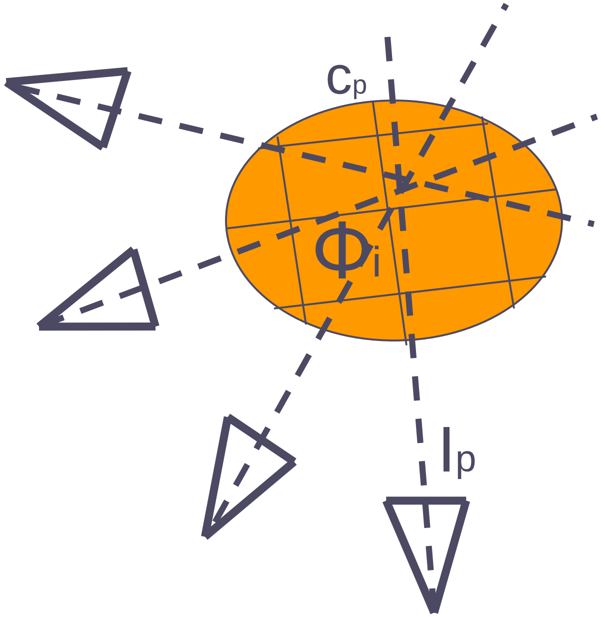


- Need several views

ART – Matrix Structure

$$I_p = \sum_i a_i \left(\int_{c_p} \phi_i \, ds \right) \qquad I = Sa$$

Basis functions $i \rightarrow$

pixels $p \downarrow$

$\int_{c_1} \phi_1 ds$	$\int_{c_1} \phi_2 ds$	$\int_{c_1} \phi_3 ds$	$\int_{c_1} \phi_4 ds$	$\int_{c_1} \phi_5 ds$	$\cdots$
$\int_{c_2} \phi_1 ds$	$\int_{c_2} \phi_2 ds$	$\int_{c_2} \phi_3 ds$	$\int_{c_2} \phi_4 ds$	$\int_{c_2} \phi_5 ds$	
$\int_{c_3} \phi_1 ds$	$\int_{c_3} \phi_2 ds$	$\int_{c_3} \phi_3 ds$	$\int_{c_3} \phi_4 ds$	$\int_{c_3} \phi_5 ds$	
$\vdots$					

invert LS in a least squares sense:

$$a = (S^T S)^{-1} S^T I$$

Frequency Space based Methods - Disadvantages

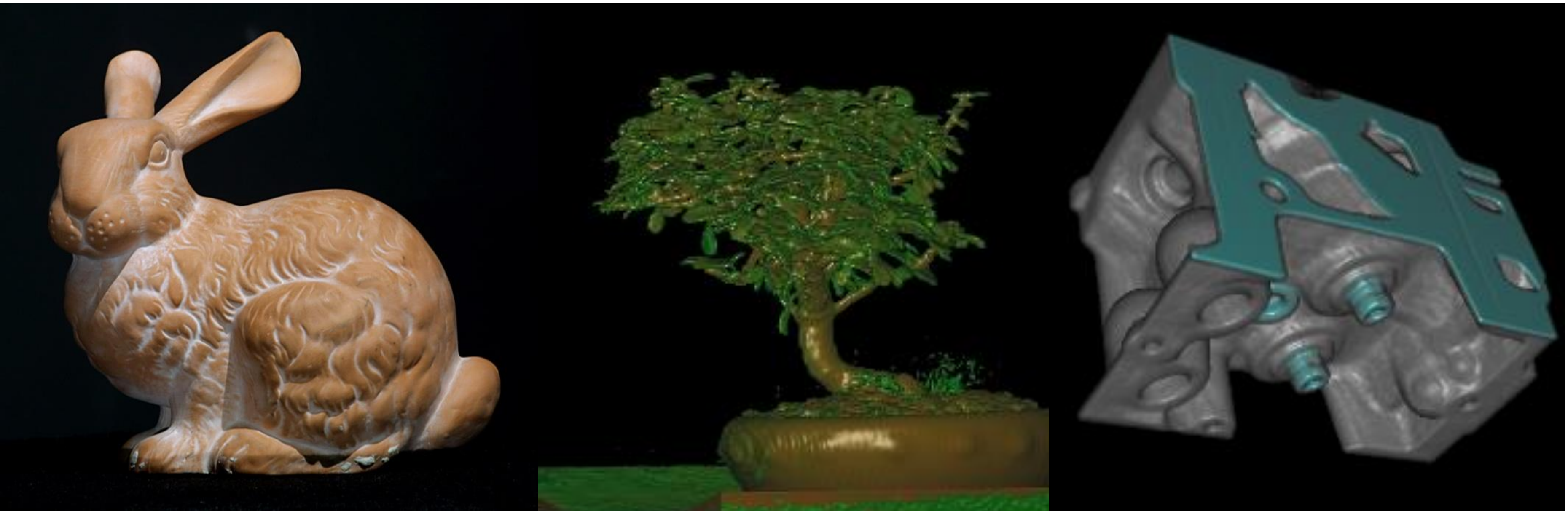
- Advantages
  - Accomodates flexible acquisition setups
  - Can be made robust to noise (next lecture)
  - Arbitrary or adaptive discretization
  - Can be implemented on GPU
- Disadvantages
  - May be slow
  - May be memory-consumptive

Volumetric 3D

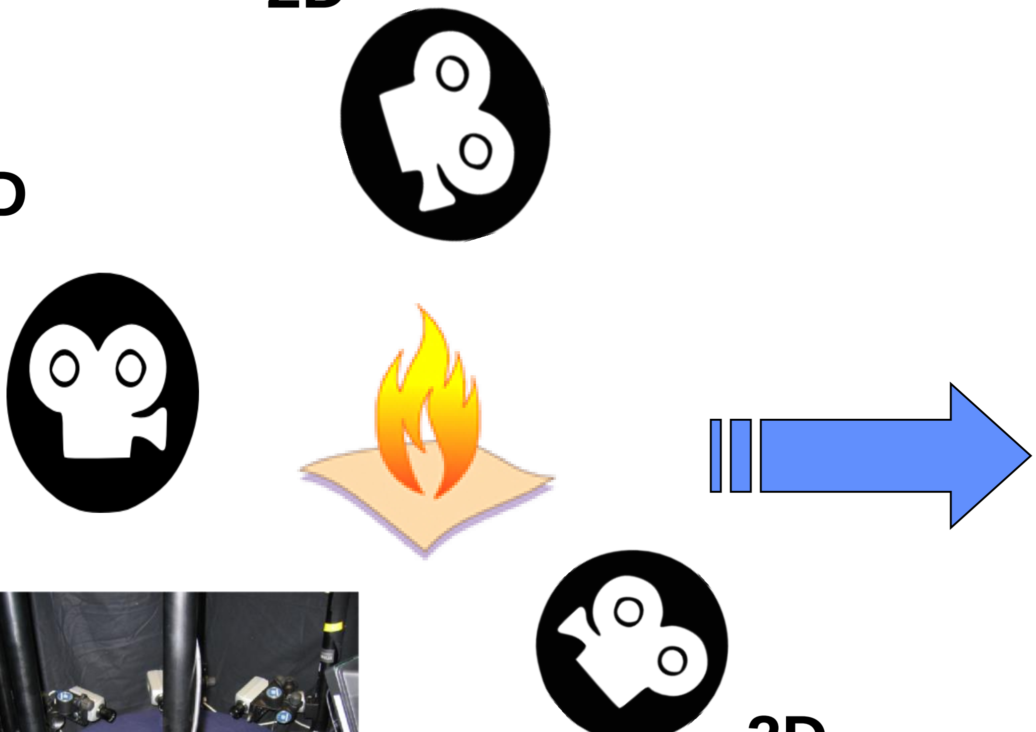
Tomography Applications

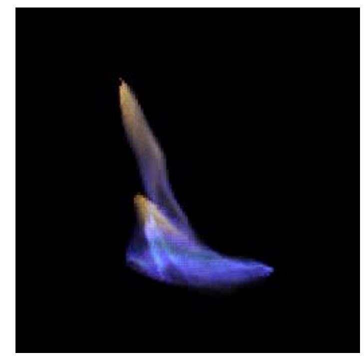
CT Applications in measurement and quality control

- Acquisition of difficult to scan objects
- Visualization of internal structures (e.g. cracks)
- No refraction



reconstruction of flames using a multi-camera setup

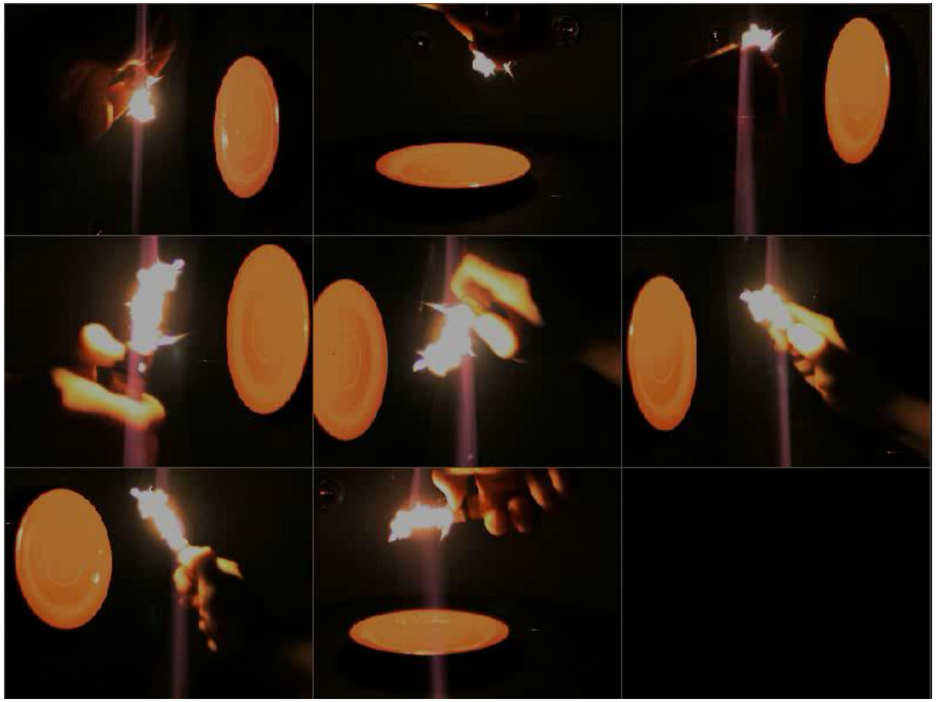




Flame tomography

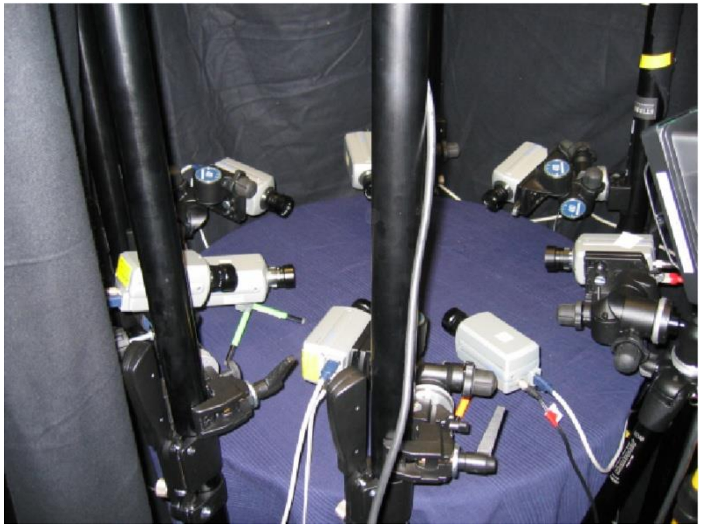
- Calibrated, synchronized camera setup
  - 8 cameras, 320 x 240 @ 15 fps

8 input views in
original camera orientation



[Ihrke' 04]

Camera setup

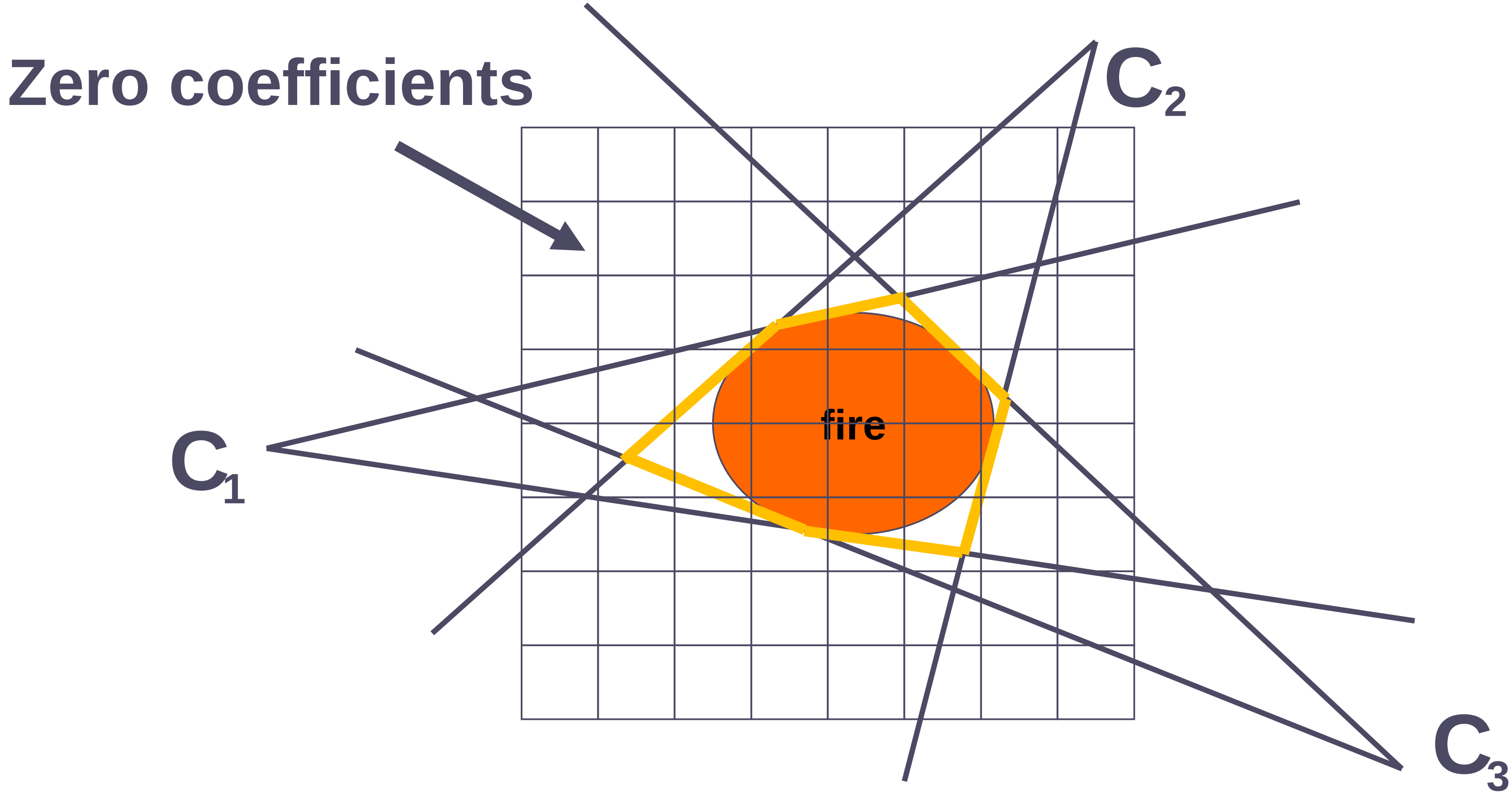


Sparse View ART - Practice

- Large number of projections is needed
- In case of dynamic phenomena
- → many cameras
  - expensive
  - inconvenient placement
- straight forward application of ART with few cameras not satisfactory

[Ihrke' 04]

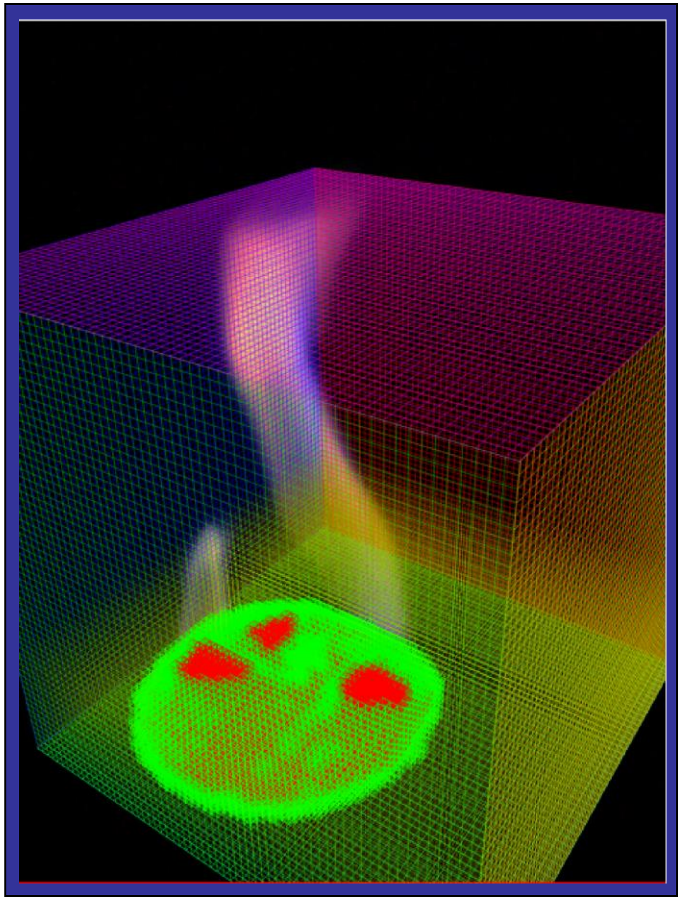
Visual Hull Restricted Tomography



[Ihrke' 04]

Visual Hull Restricted Tomography

- Only about 1/10 of the voxels contribute
- Remove voxels that do not contribute from linear system
- Complexity of inversion is significantly reduced



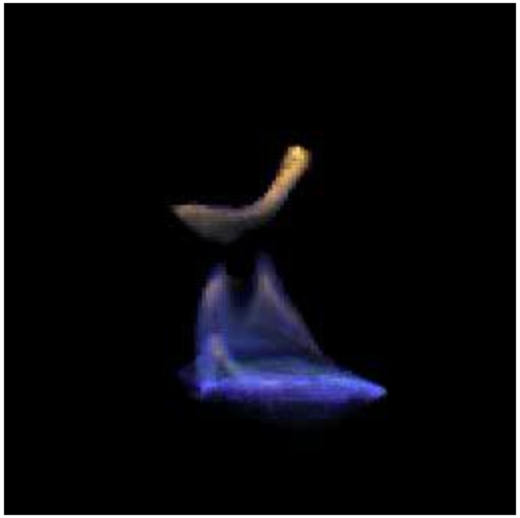
[Ihrke' 04]

Animated Flame Reconstruction

[Ihrke' 04]

- 

  frame 86

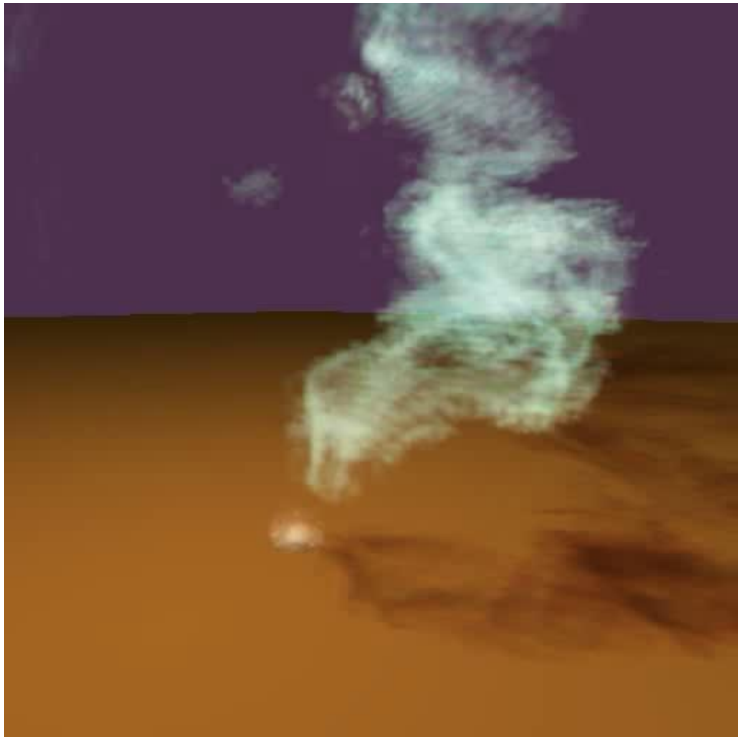
  

  frame 194

animated reconstructed flames

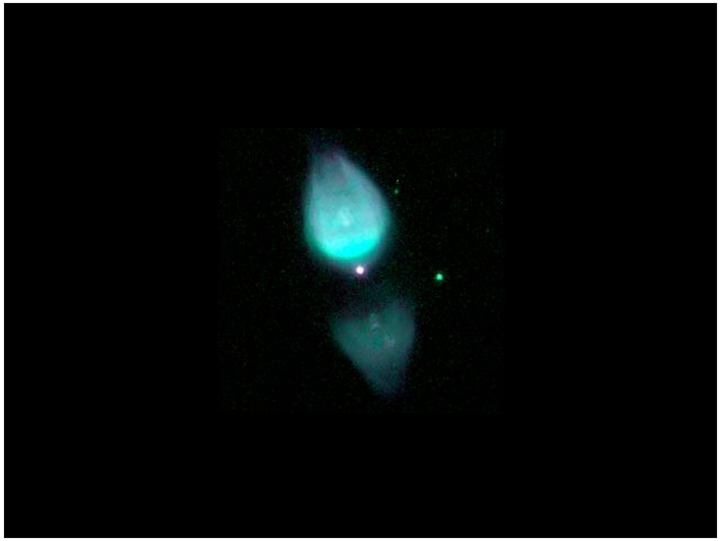
Smoke Reconstructions

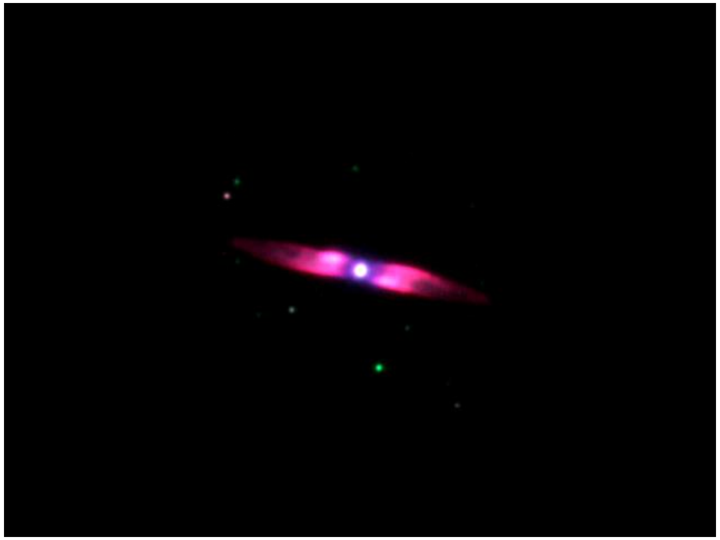
[Ihrke' 06]



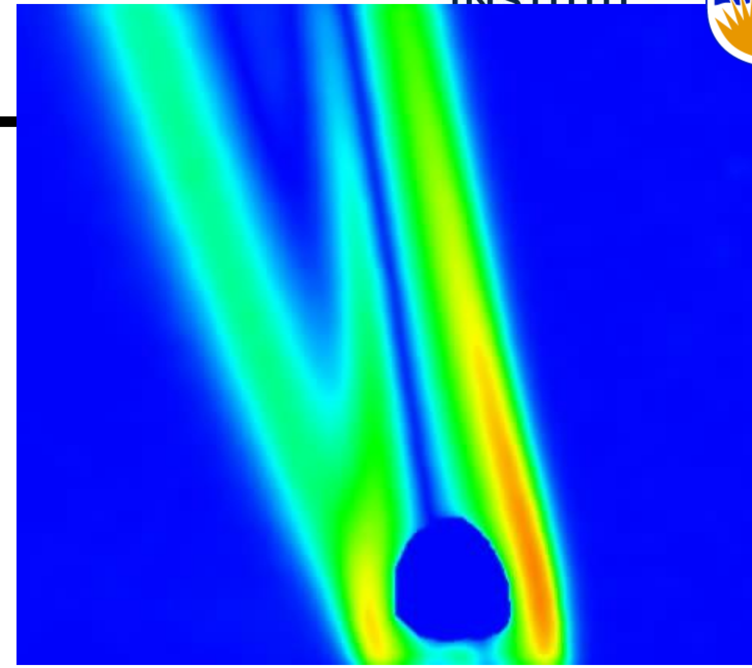
3D Reconstruction of Planetary Nebulae

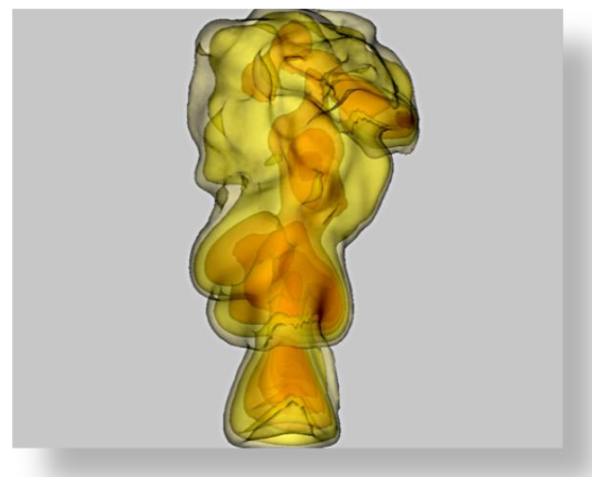
- only one view available **[Magnor04]**
- exploit axial symmetry
- essentially a 2D problem



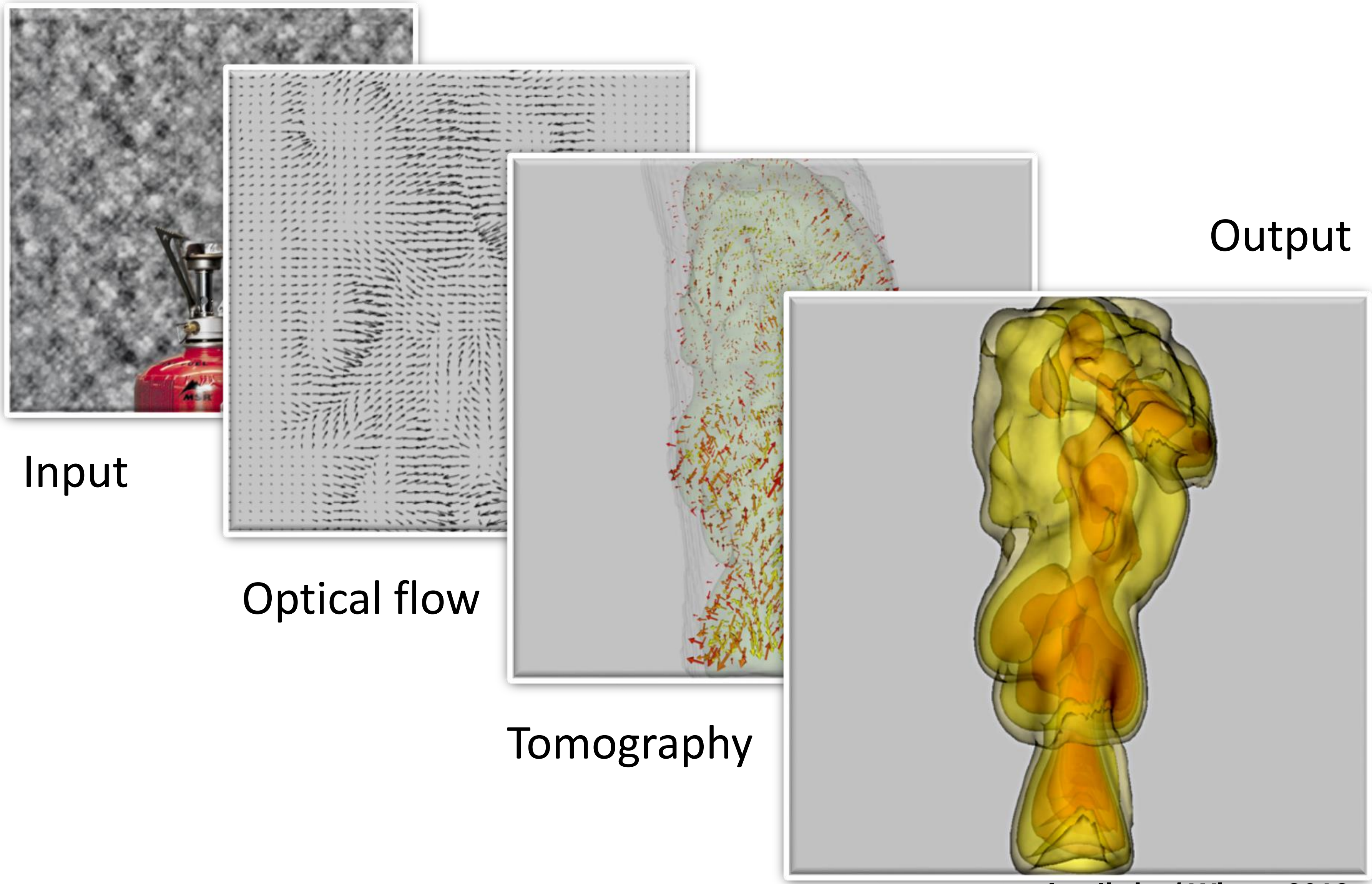



HL





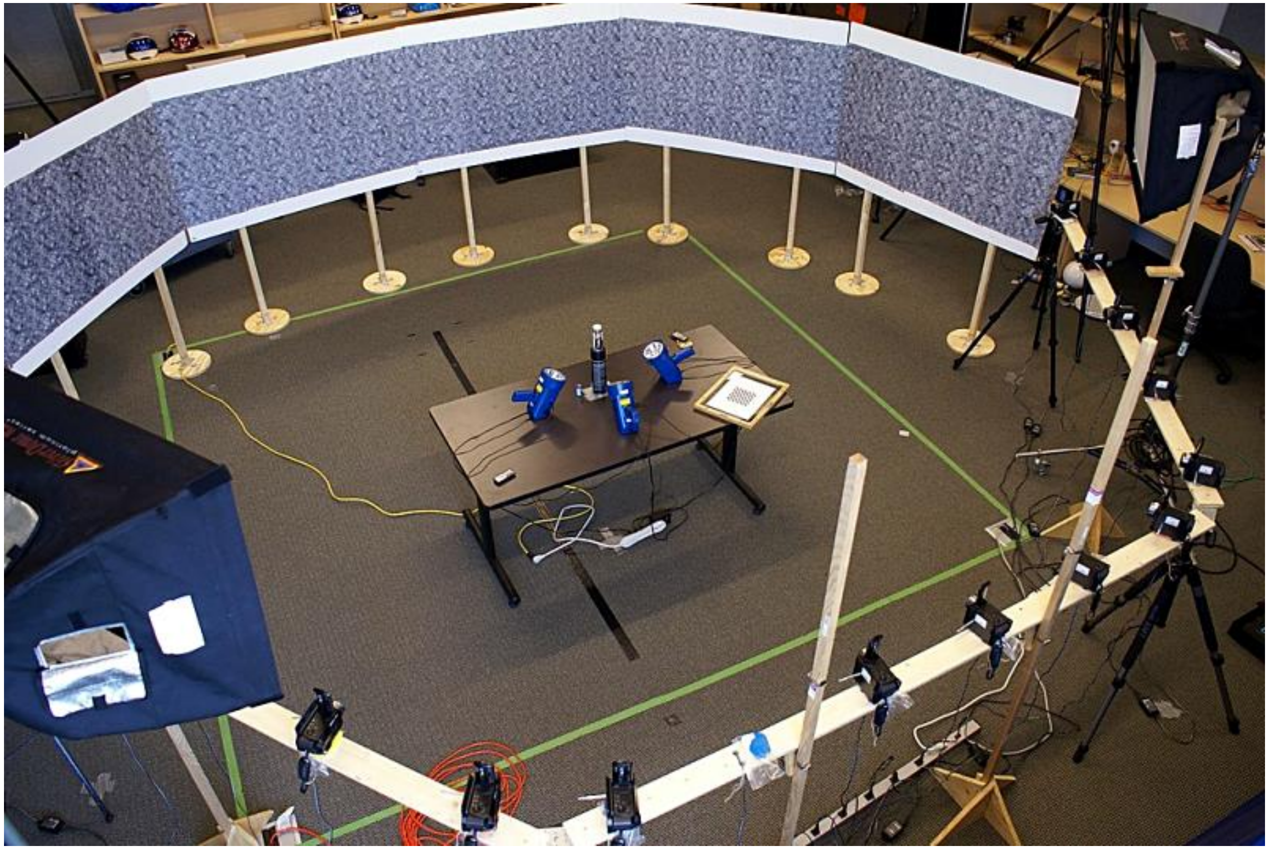
Schlieren Tomography



Schlieren Tomography - Acquisition

16 camera array (consumer camcorders)

Synchronization & rolling shutter compensation



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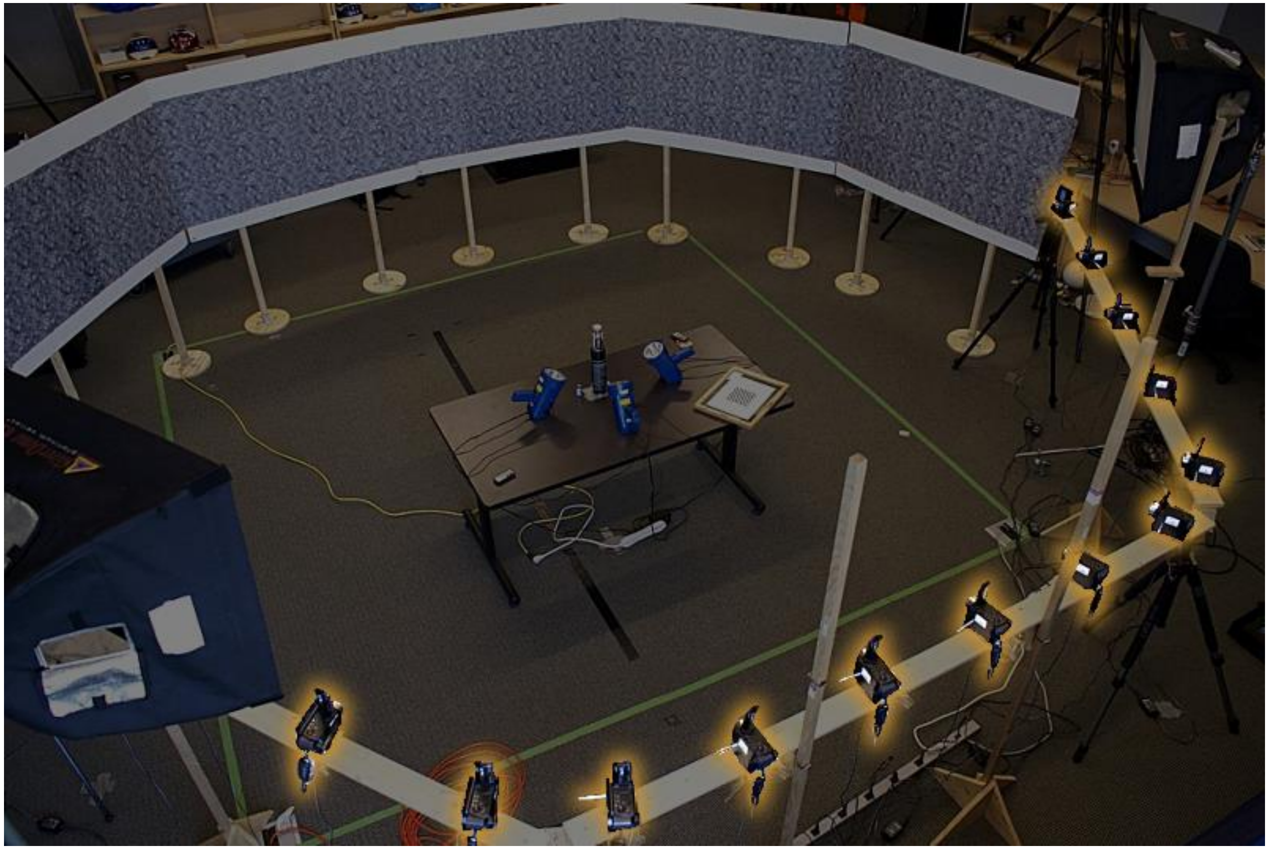

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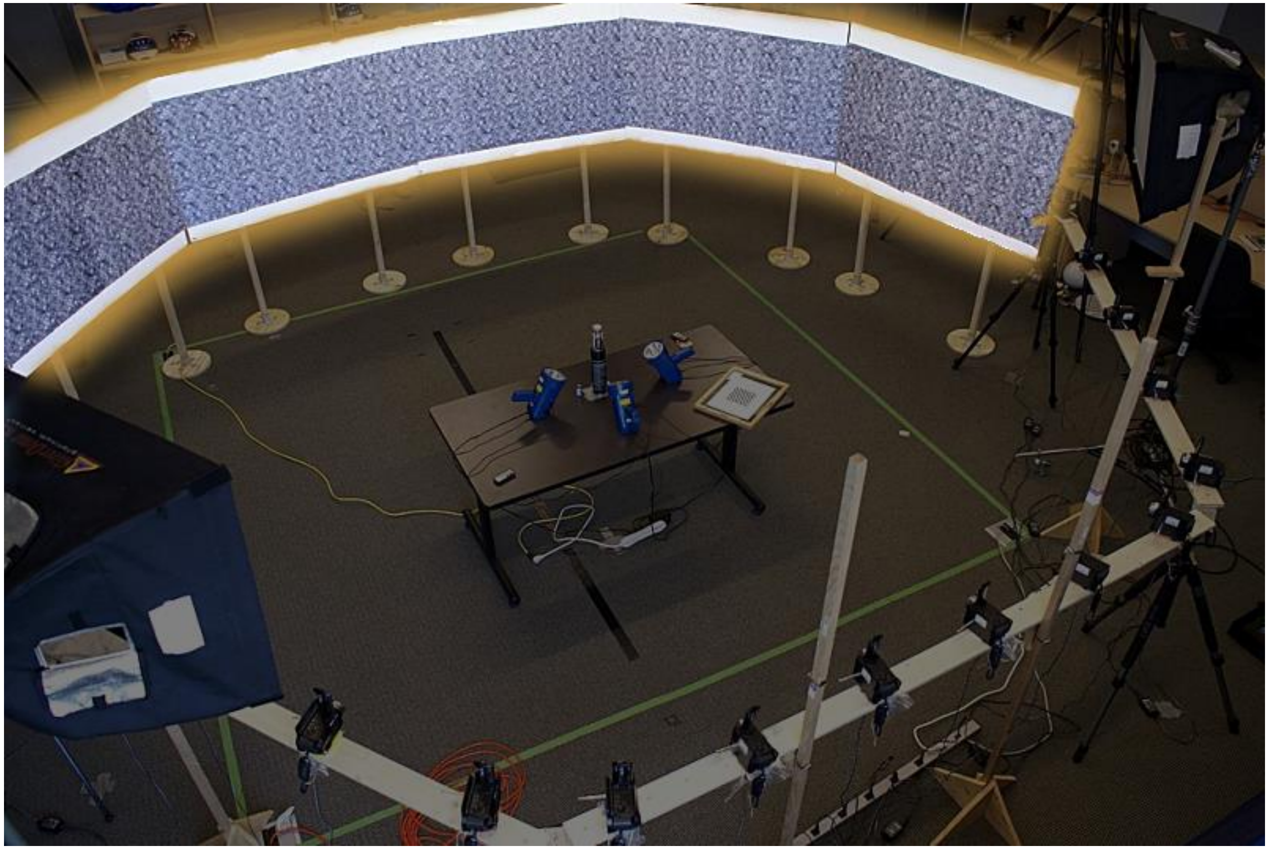

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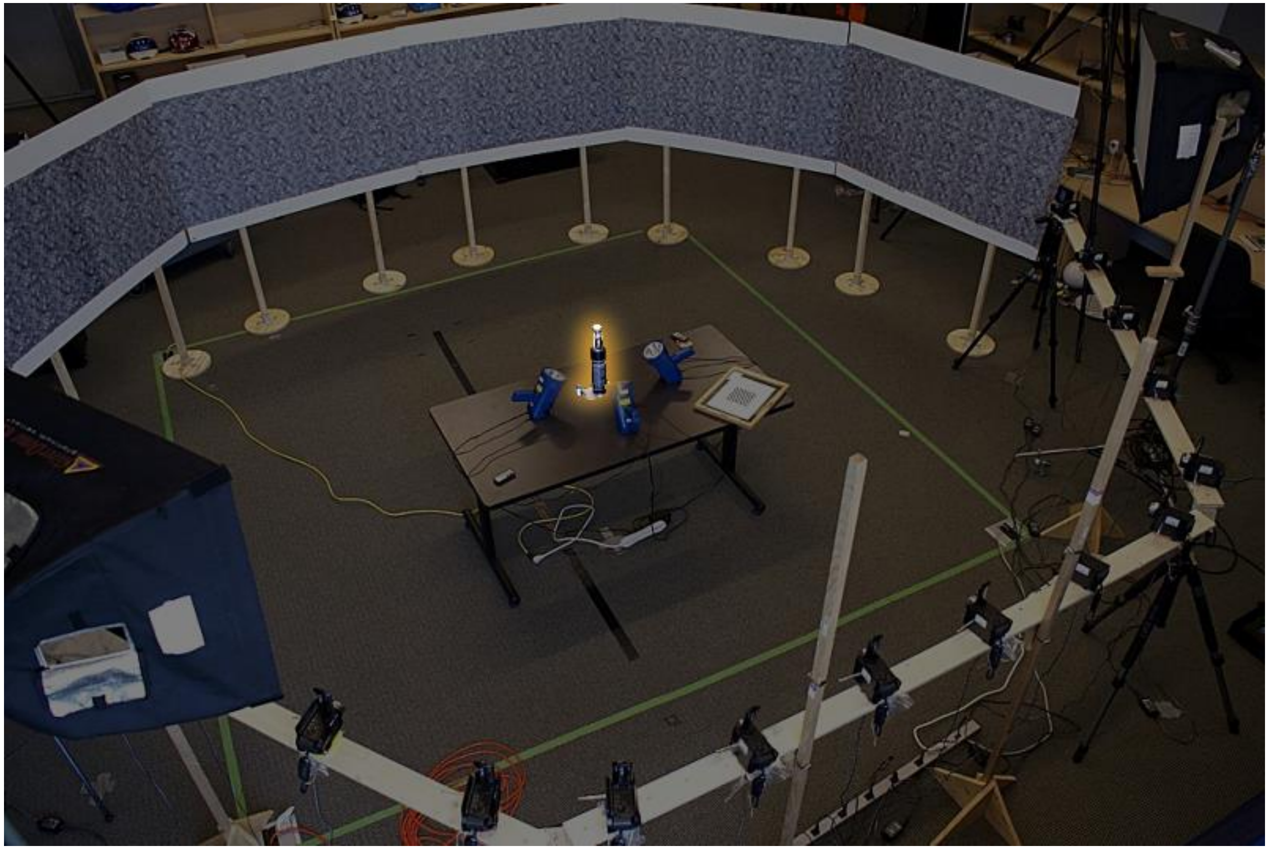
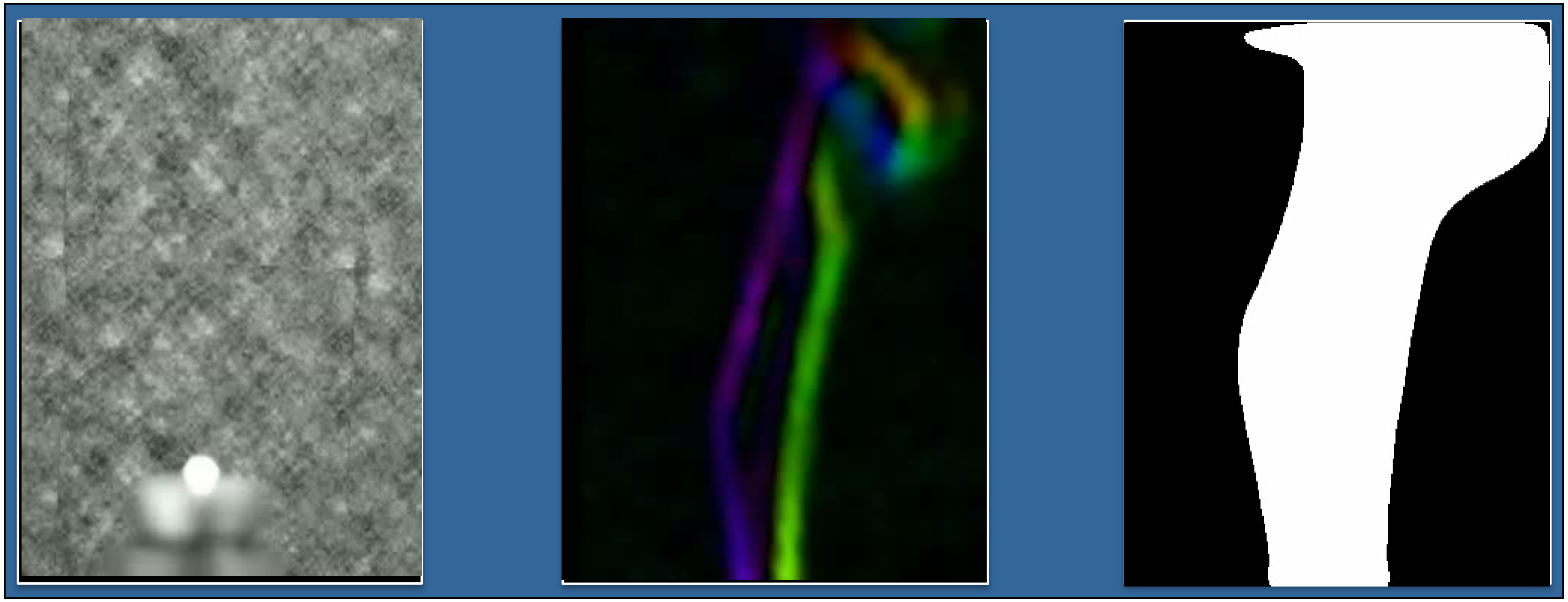


Schlieren CT – Image Processing



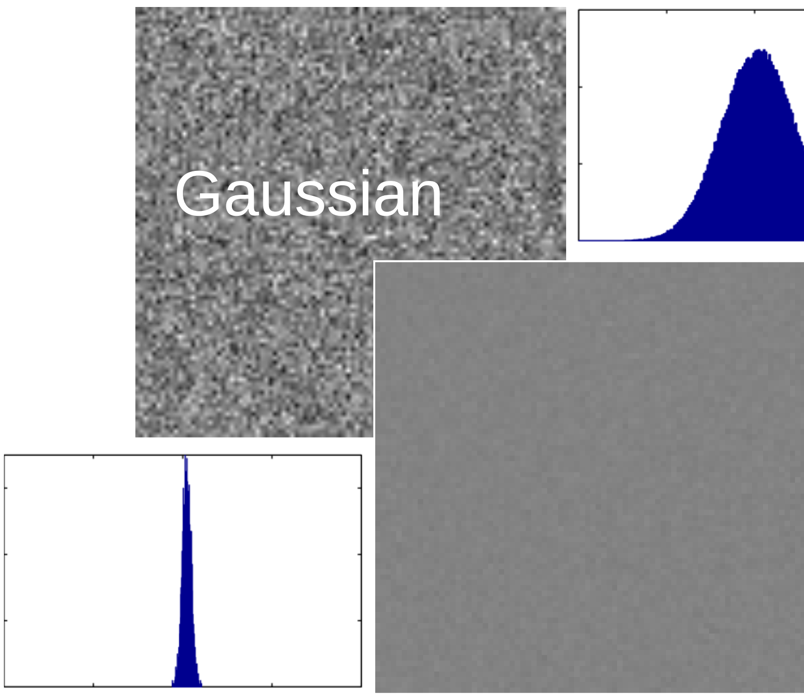
Input Optical flow Mask

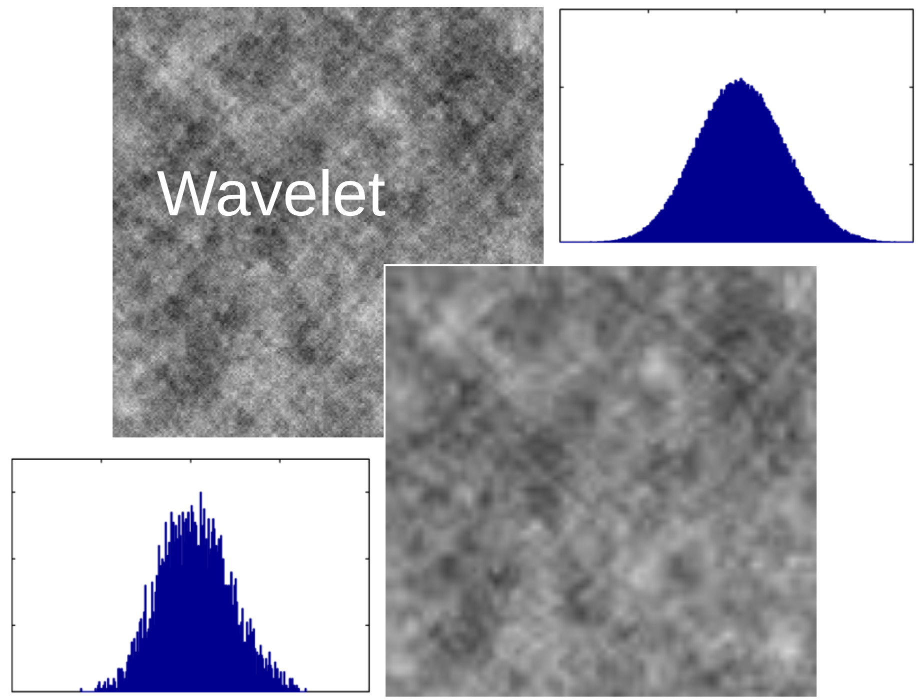
Schlieren CT – Background Pattern

High frequency detail everywhere

Decouple pattern resolution from sensor

Wavelet noise [Cook 05]





Schlieren CT - Image Formation

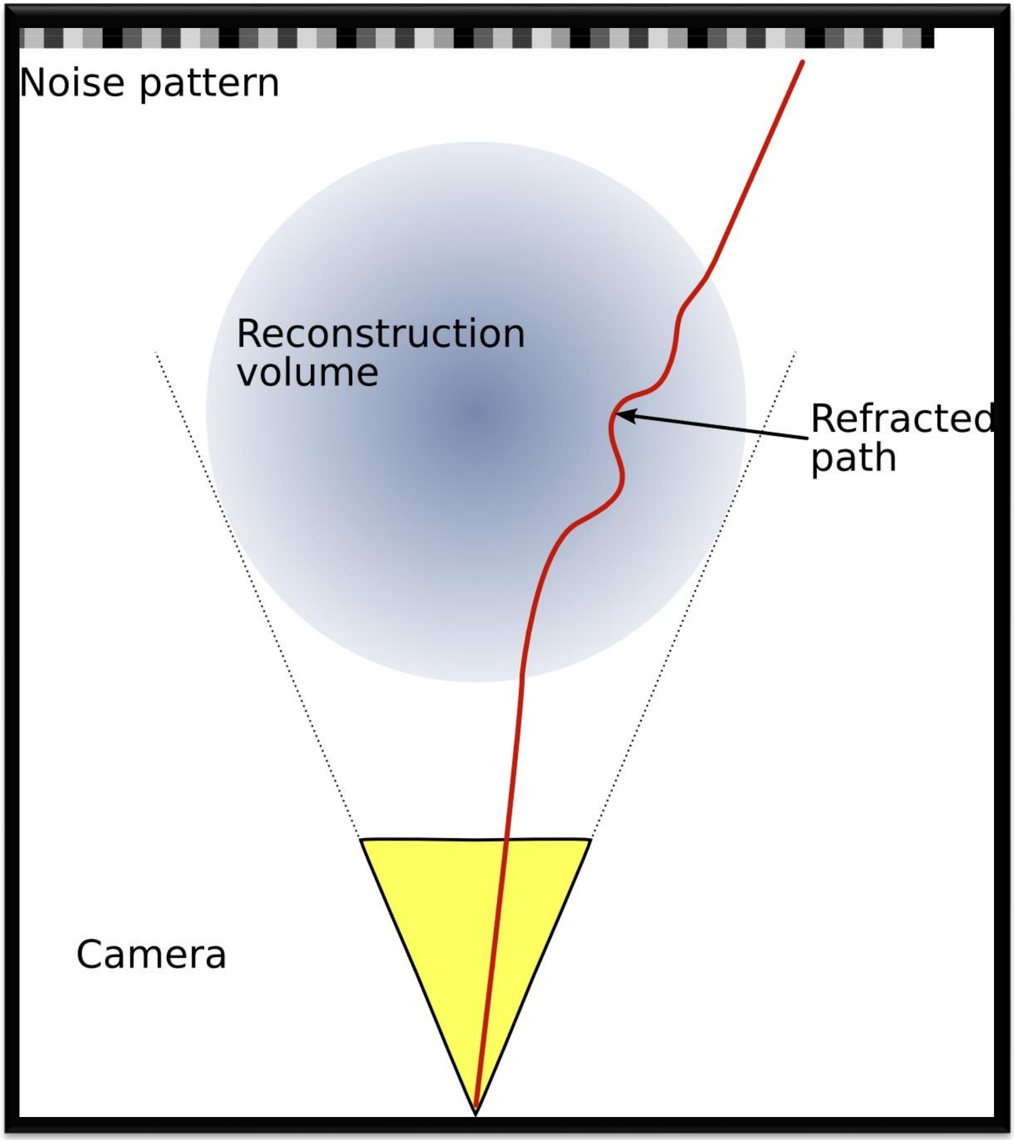
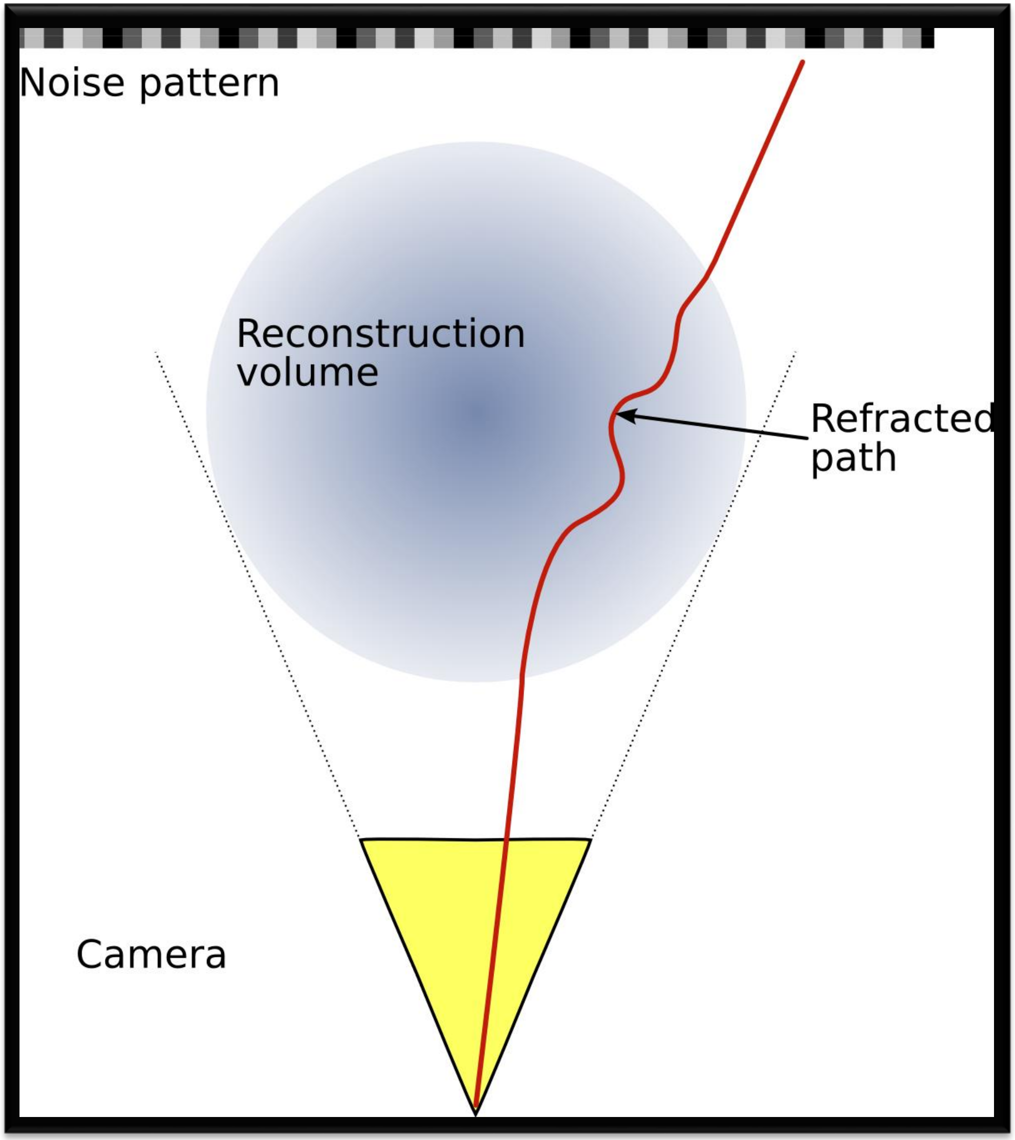


Image formation in continuously refracting media

Curved Rays

Described well by Ray Equation of Geometric Optics

Schlieren CT - Image Formation



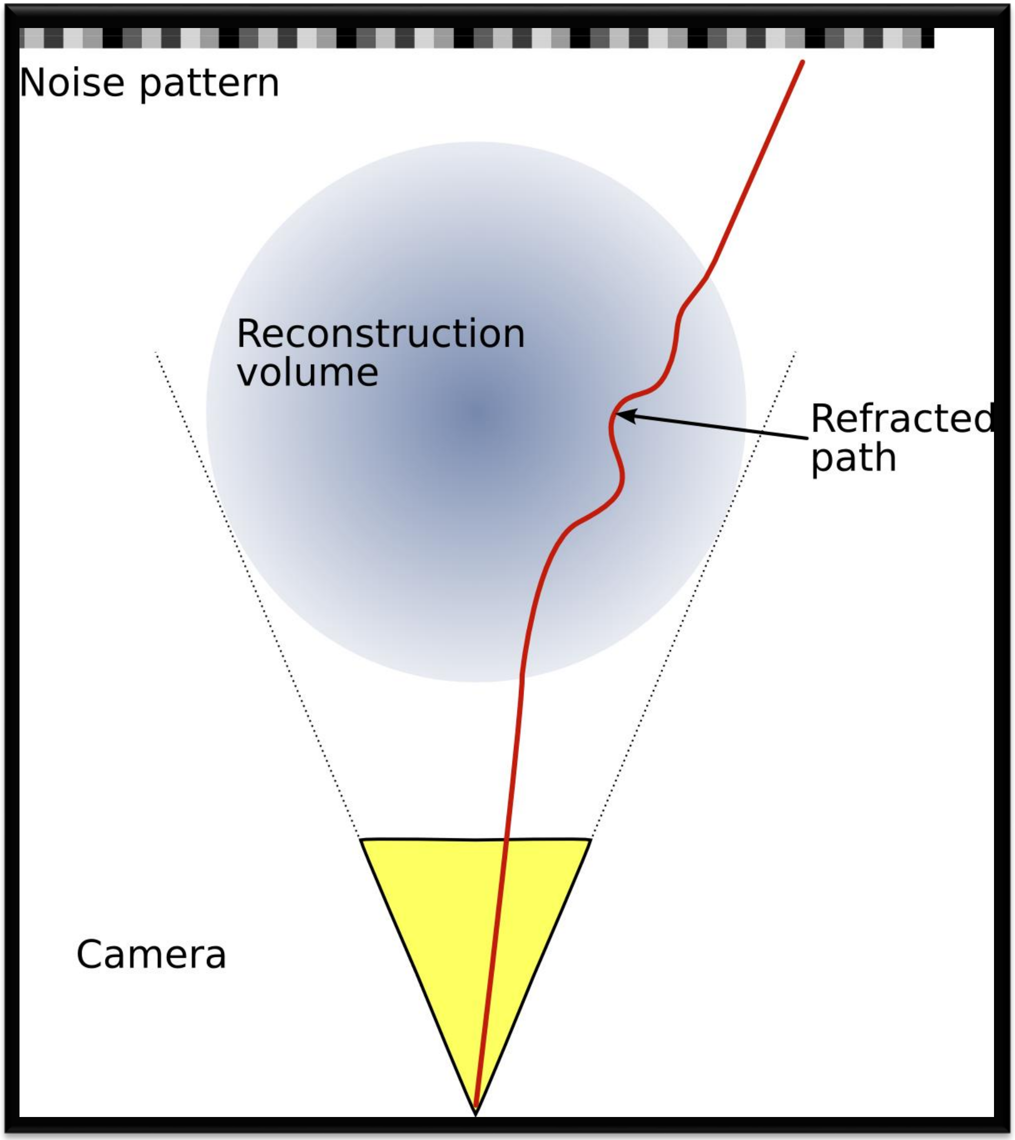
Continuous ray tracing, e.g. [Stam 96, Ihrke 07]

Set of 1st order ODE's :

$$n\frac{d\mathbf{x}}{ds} = \mathbf{d}$$

$$\frac{d\mathbf{d}}{ds} = \nabla n$$

Schlieren CT - Ray equation



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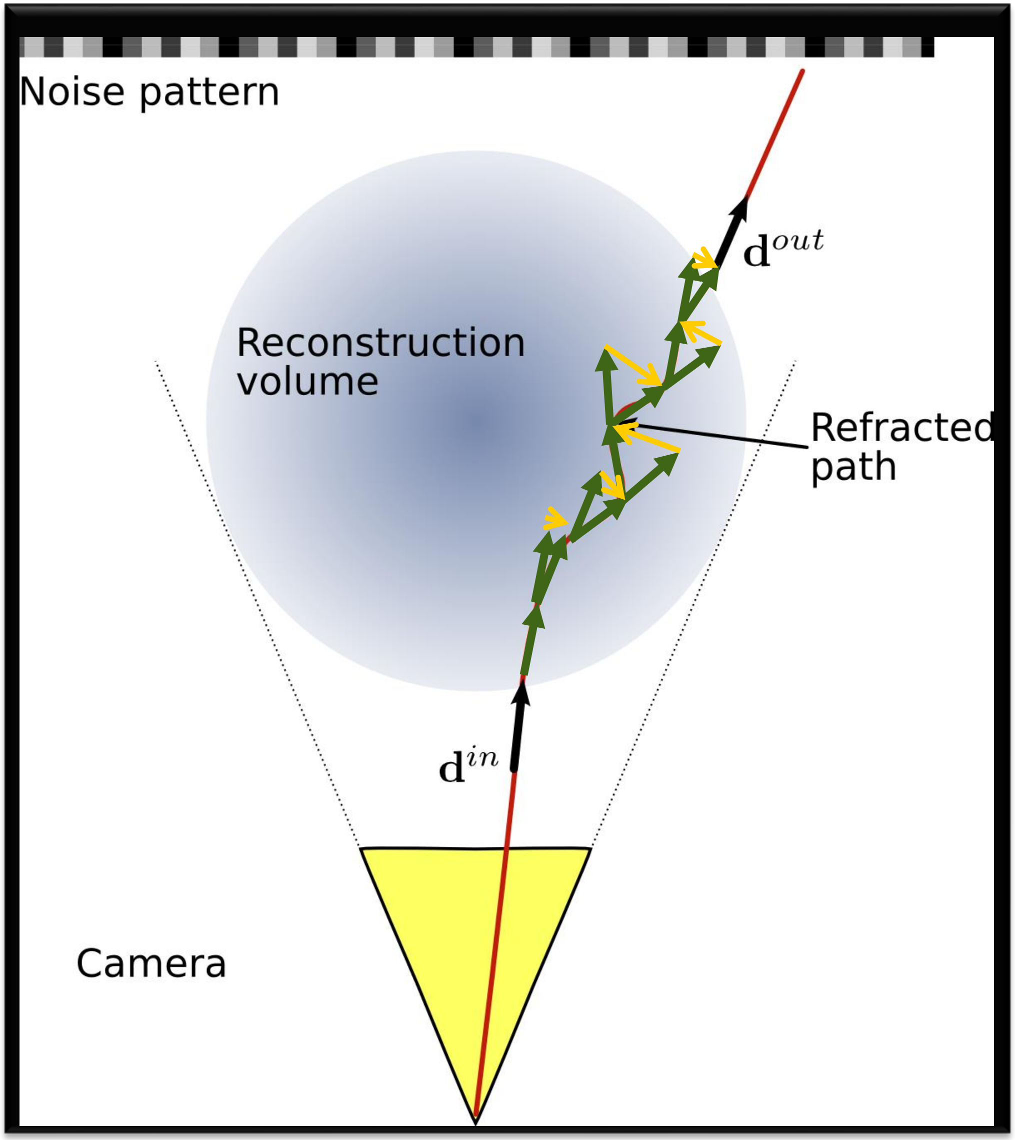
Schlieren CT - Ray equation



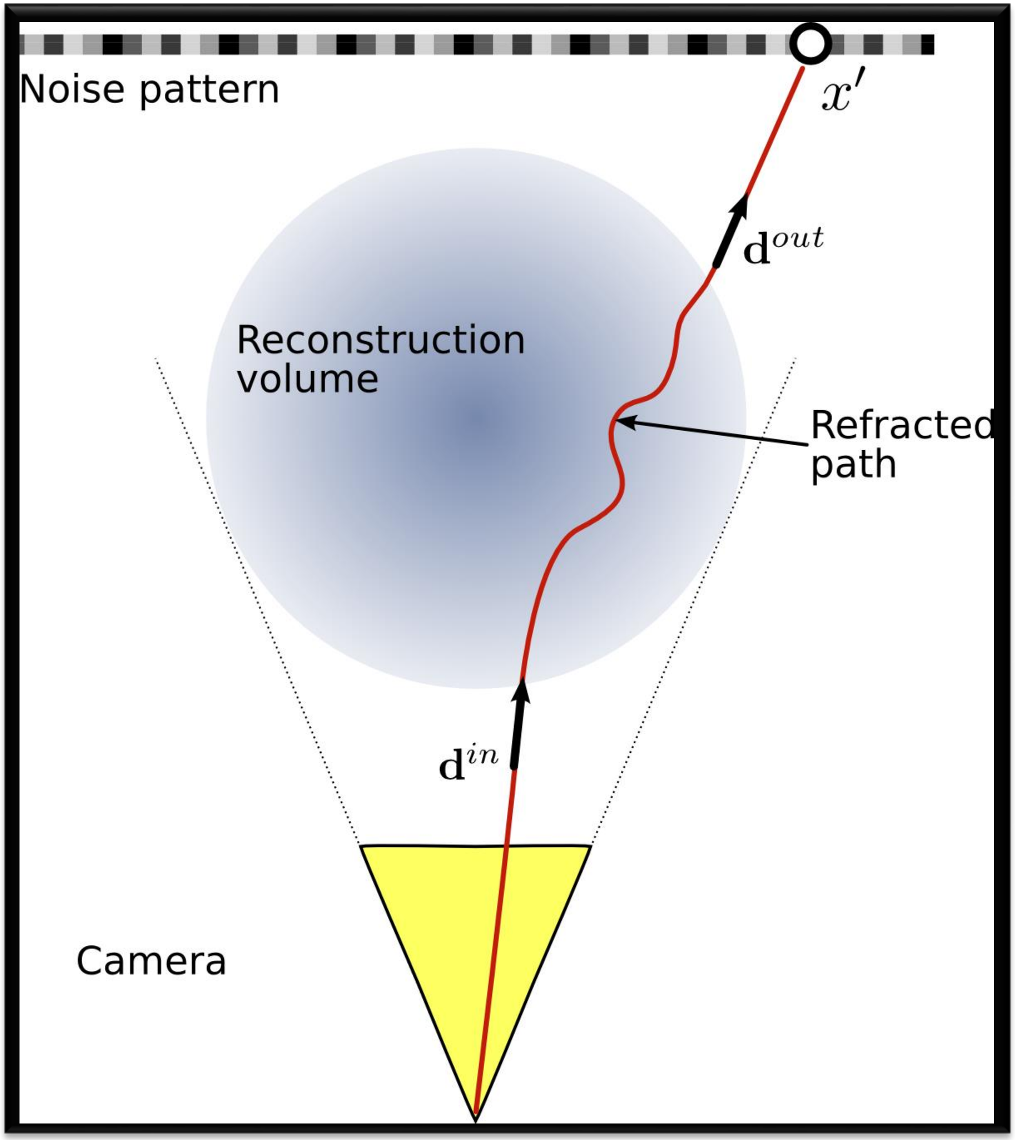
Integrating

$$\frac{d\mathbf{d}}{ds} = \nabla n$$

yields

$$\mathbf{d}^{out} = \mathbf{d}^{in} + \int_c \nabla n \, ds$$

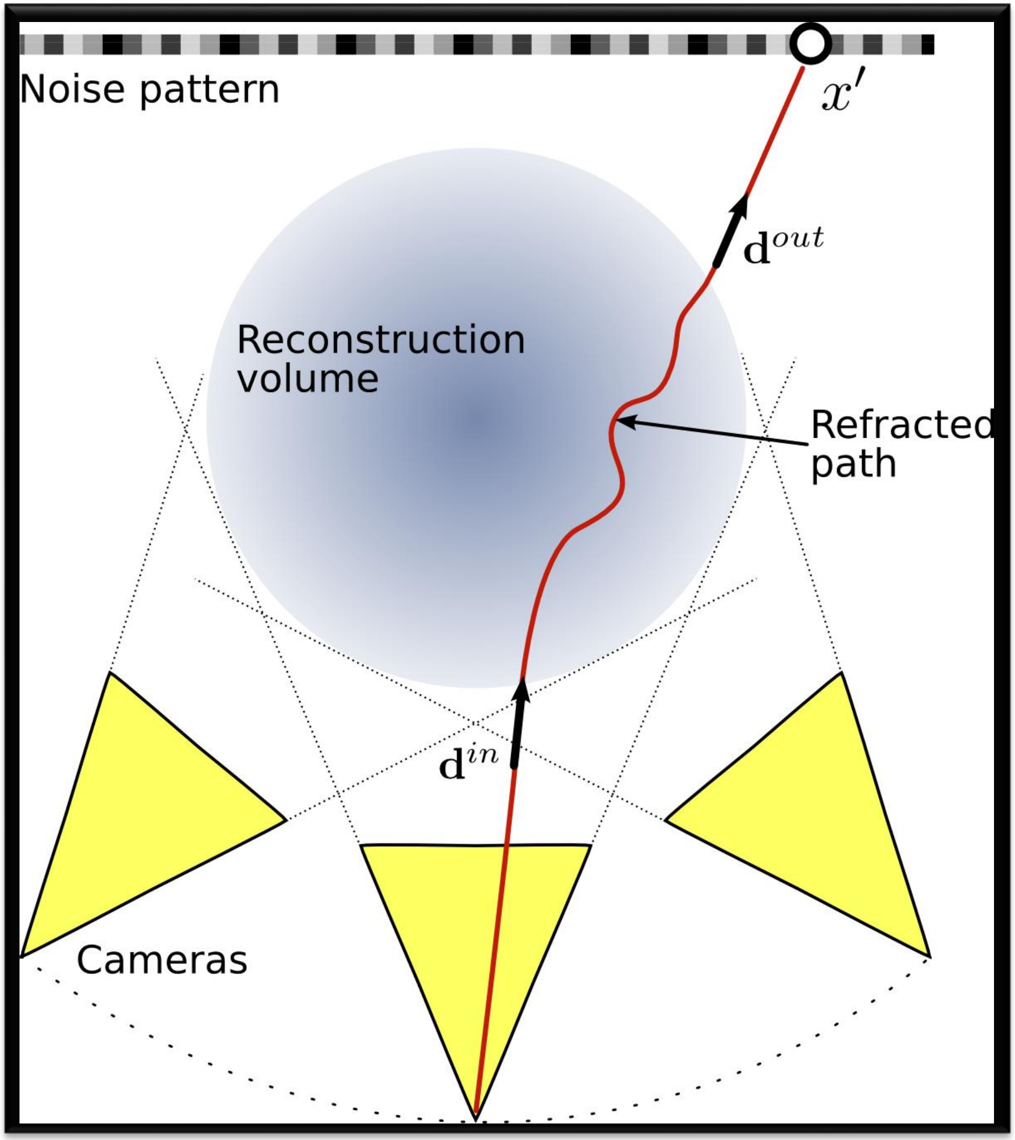
Schlieren Tomography



Basic equation for Schlieren Tomography

$$\mathbf{d}^{out} - \mathbf{d}^{in} = \int_c \nabla n \, ds$$

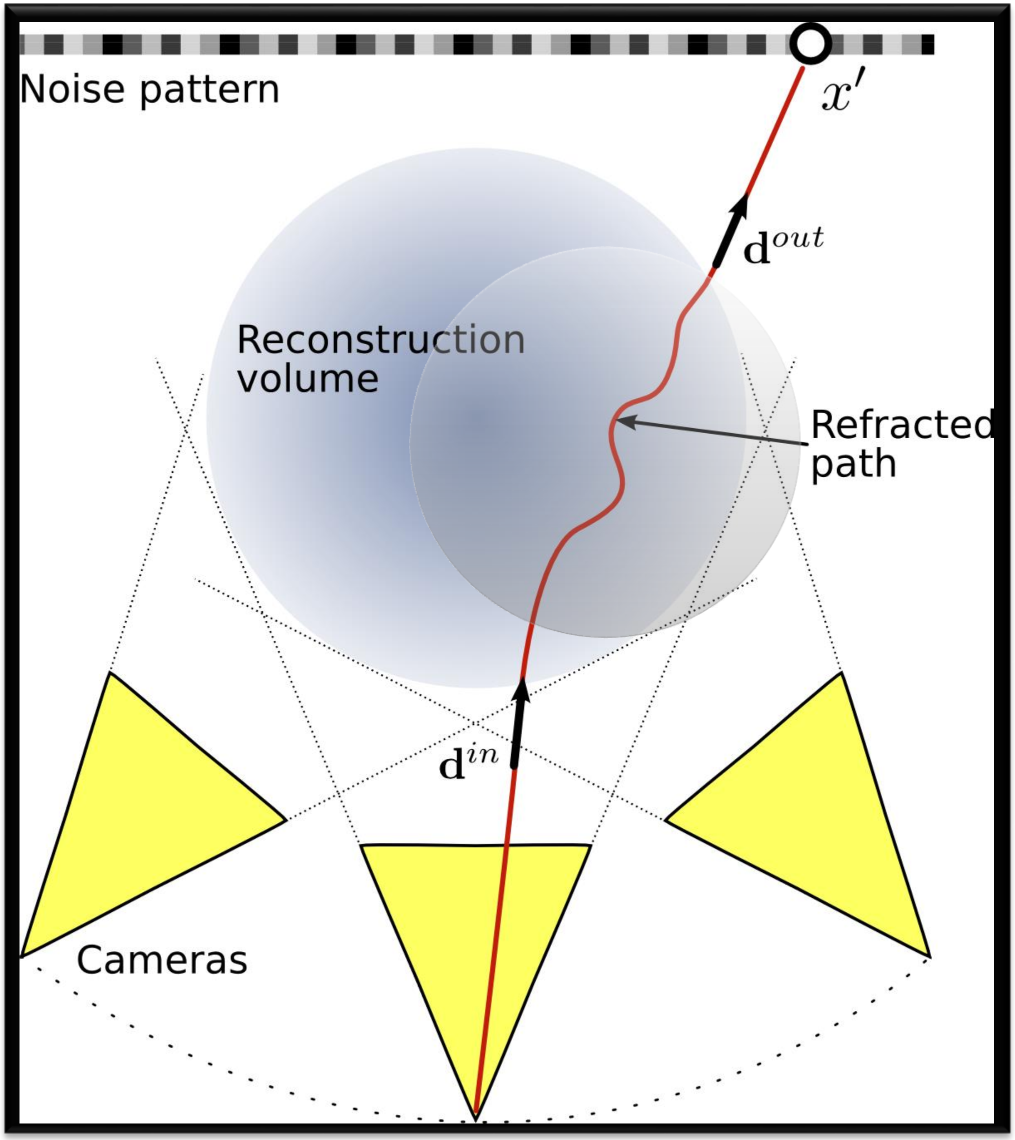
Schlieren Tomography



Based on measurements of line integrals from different orientations

$$\mathbf{d}^{out} - \mathbf{d}^{in} = \int_c \nabla n \, ds$$

Schlieren Tomography



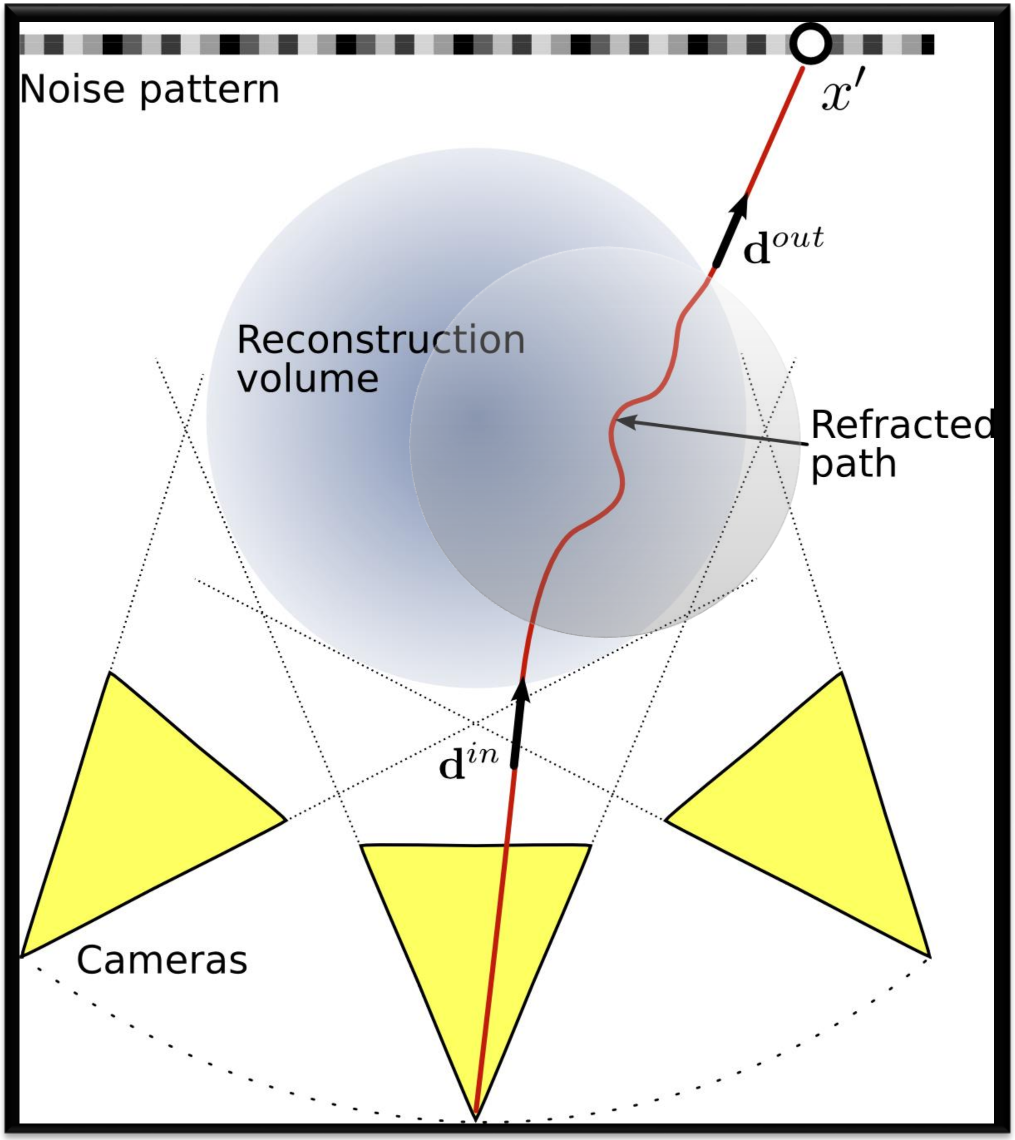
Ray path must be known

BUT: unknown refractive index

In practice, ray bending negligible [Venkatakrishnan'04]

$$\mathbf{d}^{out} - \mathbf{d}^{in} = \int_c \nabla n \, ds$$

Schlieren Tomography



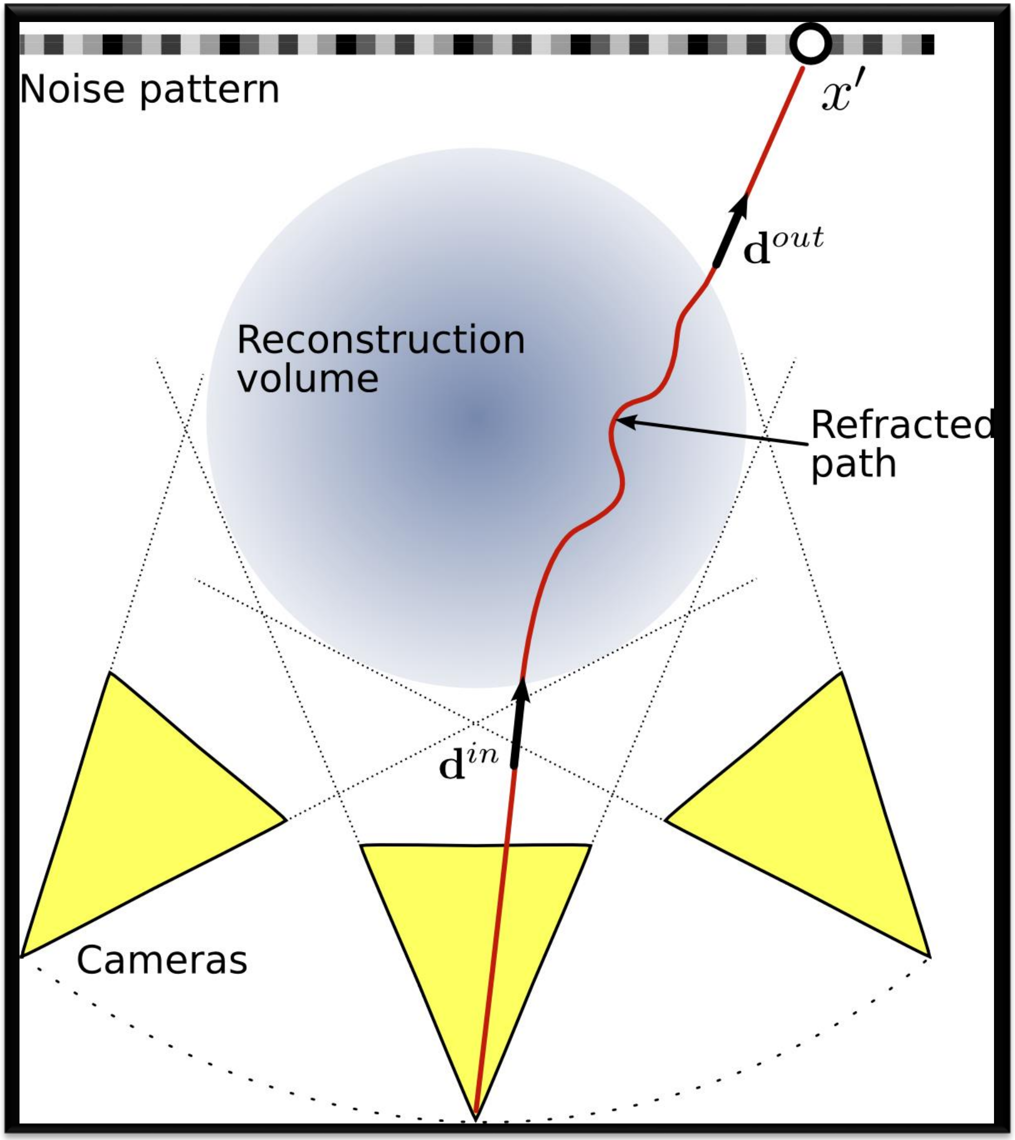
Ray path must be known

BUT: unknown refractive index

Affects integration path only, equation still holds approximately!

$$\mathbf{d}^{out} - \mathbf{d}^{in} = \int_c \nabla n \, ds$$

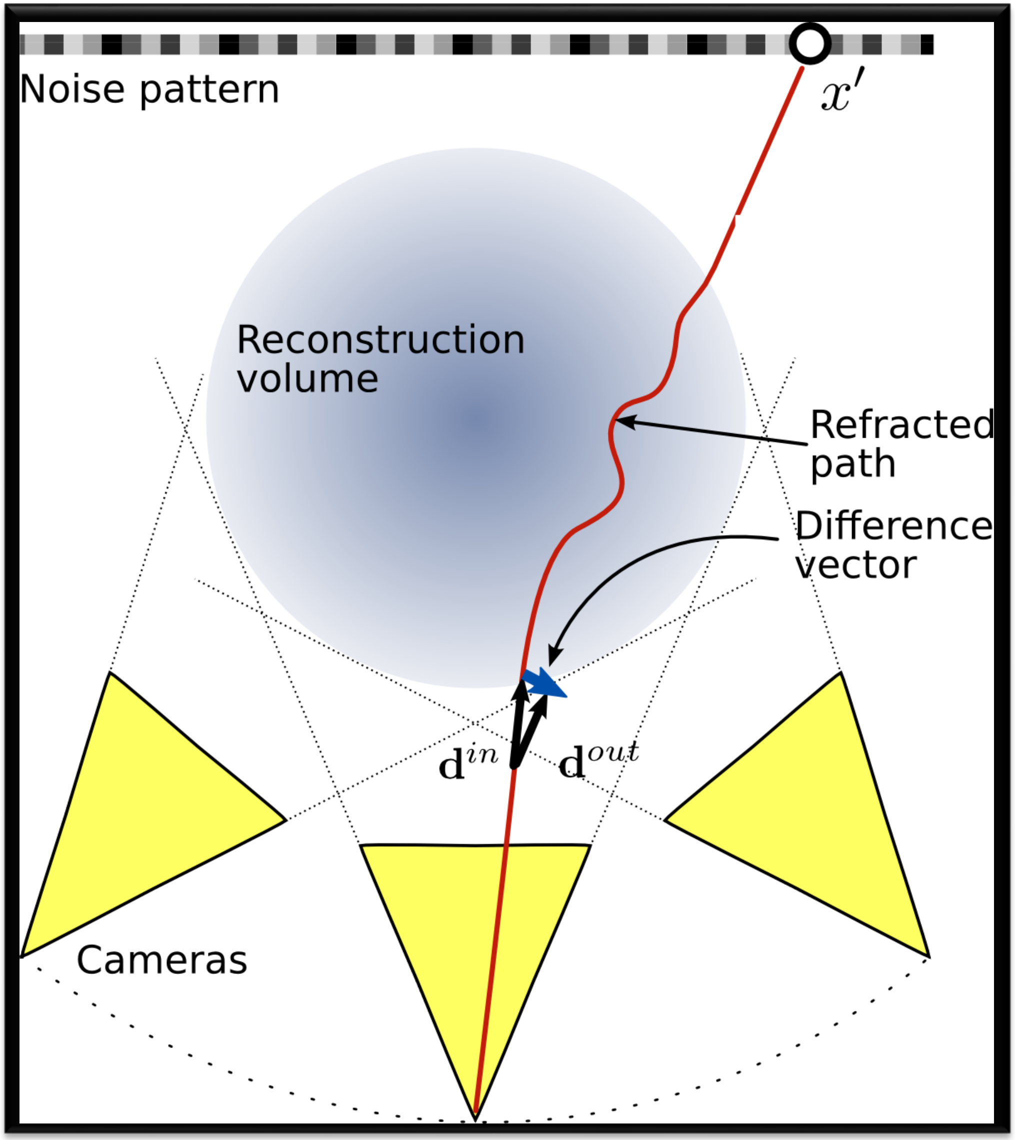
Schlieren Tomography - Measurements



Measure difference vector

$$\mathbf{d}^{out} - \mathbf{d}^{in} = \int_c \nabla n \, ds$$

Schlieren Tomography - Measurements



Measure difference vector

Component parallel to optical axis is lost

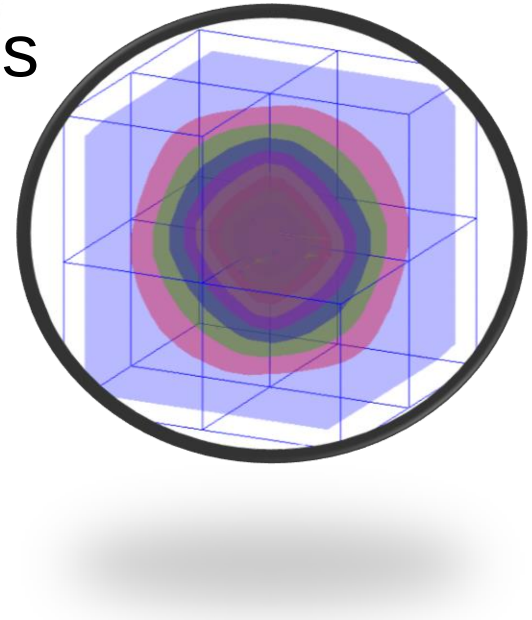
$$\mathbf{d}^{out} - \mathbf{d}^{in} = \int_c \nabla n \, ds$$

Schlieren Tomography –Linear System

Vector-valued tomographic problem

Discretize gradient

- Radially symmetric basis functions



$$\overline{\nabla n} = \sum_i \mathbf{n}_i \phi_i$$

Linear system in

$$\overline{\mathbf{d}}^{out} - \mathbf{d}^{in} = \int_c \sum_i \mathbf{n}_i \phi_i \, ds = \sum_i \mathbf{n}_i \int_c \phi_i \, ds$$

Schlieren Tomography - Integration

Given ∇n from tomography

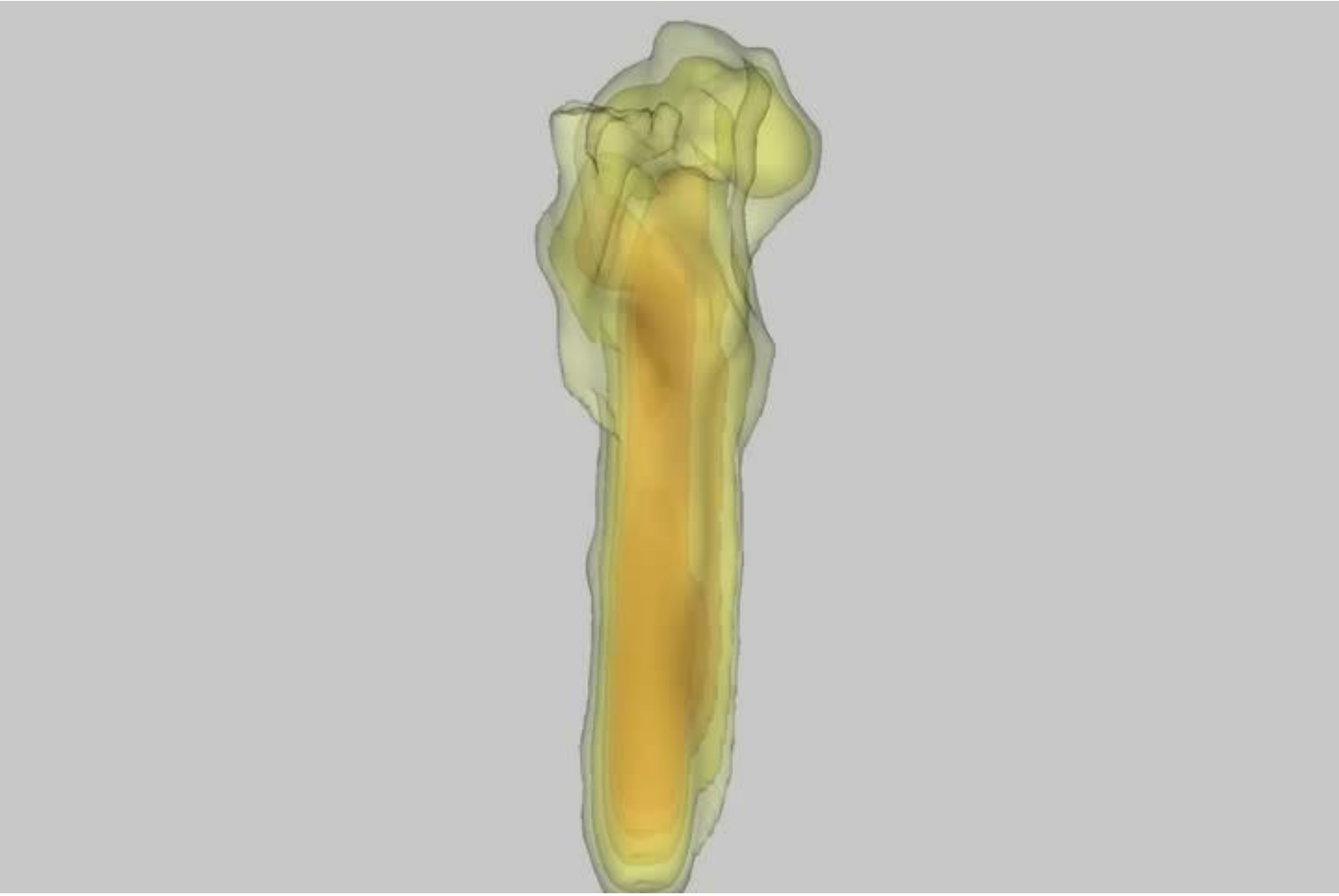
Compute n from definition of Laplacian

$$\nabla \cdot \nabla n = \Delta n$$

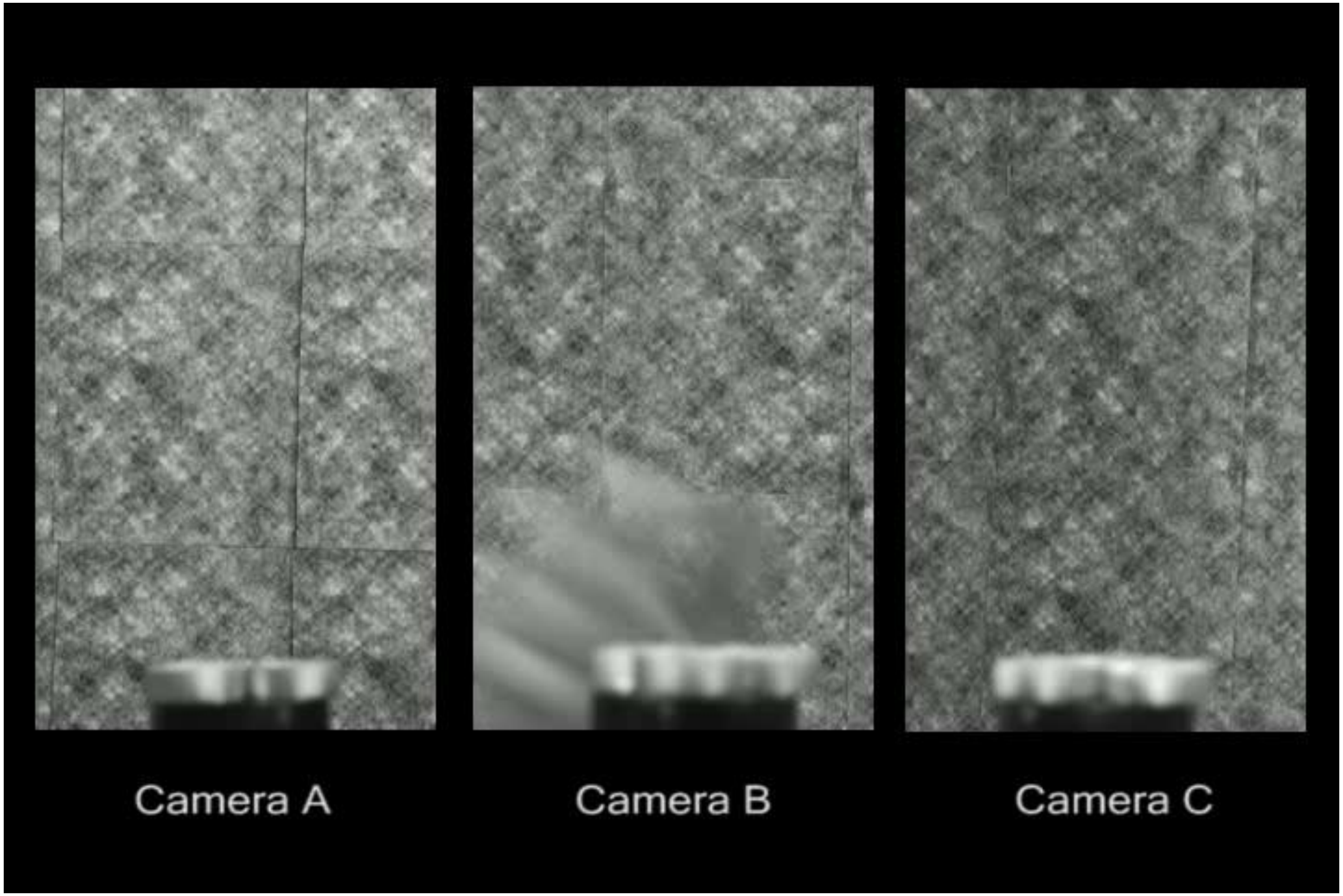
Solve Poisson equation to get refractive index

- Inconsistent gradient field due to noise and other measurement error
- Anisotropic diffusion

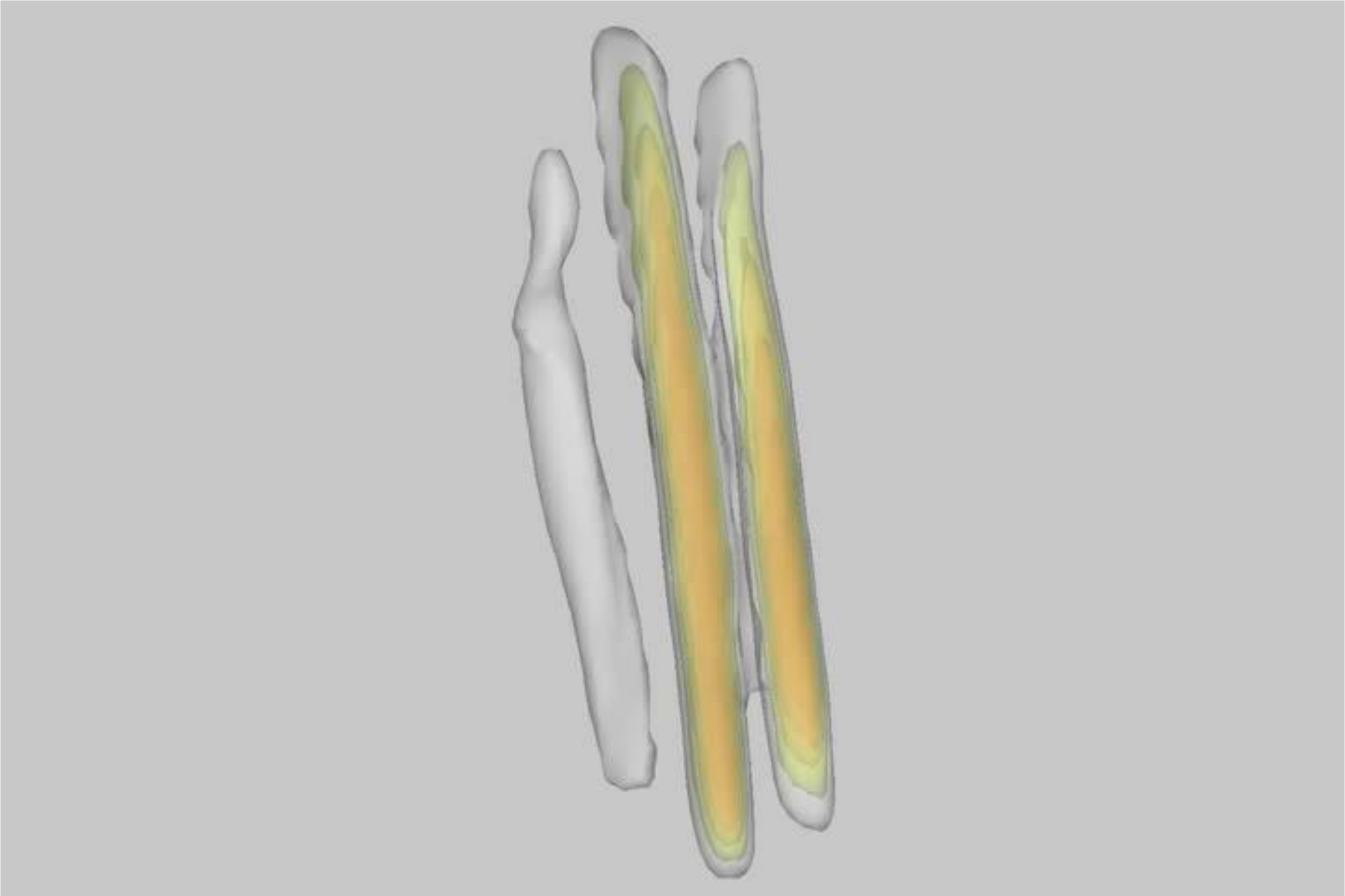
Schlieren Tomography - Results



Schlieren Tomography - Results



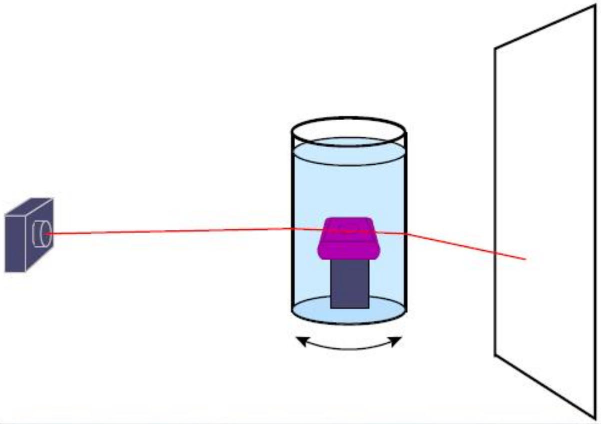
Schlieren Tomography - Results

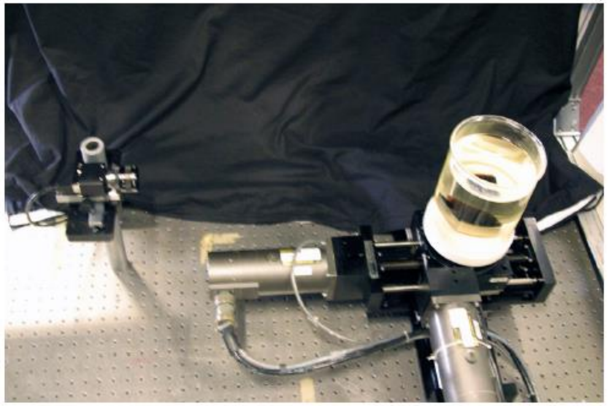


3D Scanning of Glass Objects [Trifonov06]

- visible light tomography of glass objects
  - needs straight ray pathes
- compensate for refraction
  - immerse glass object in water
  - add refractive index matching agent

  → "ray straightening"
- apply tomographic reconstruction





3D Scanning of Glass Objects [Trifonov06]

- Tomographic reconstruction results in volume densities
- use marching cubes to extract object surfaces