

Computational Optical Imaging - Optique Numerique

-- Multiple View Geometry and Stereo --

Winter 2013

Ivo Ihrke

with slides by Thorsten Thormaehlen

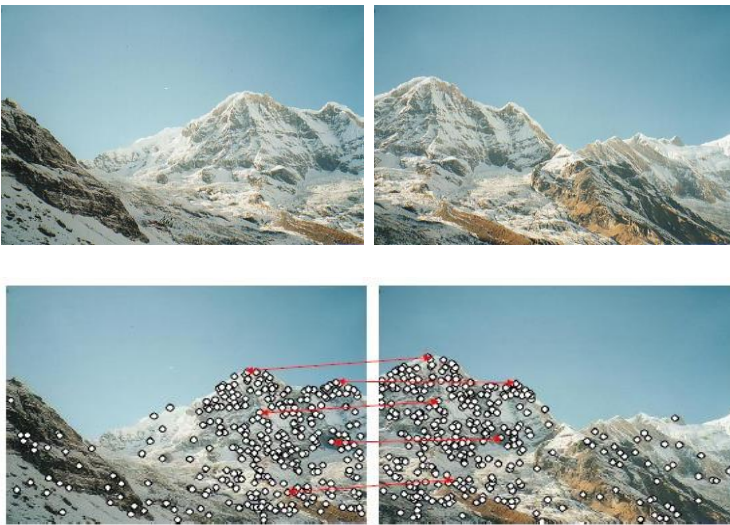
Feature Detection and Matching

Wide-Baseline-Matching

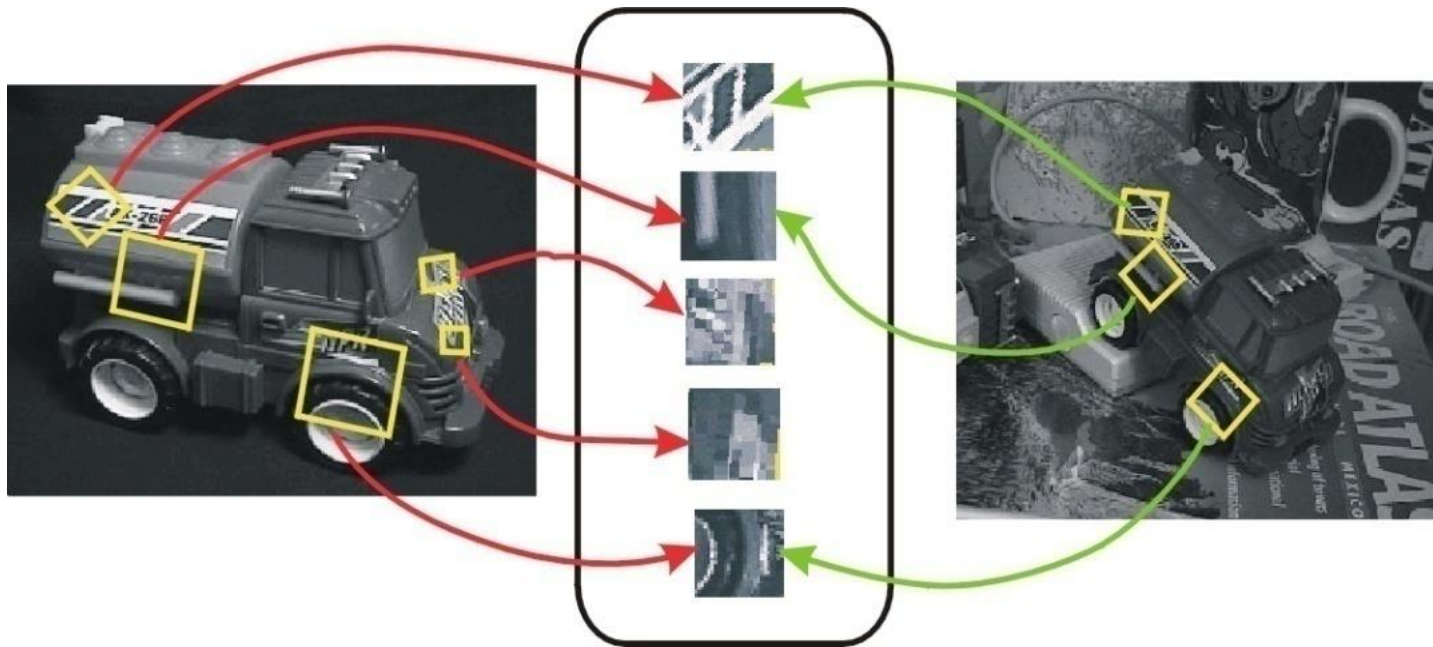
- SIFT = Scale Invariant Feature Transform
 - David G. Lowe: “Distinctive image features from scale-invariant keypoints” (IJCV 2004)
 - <http://www.cs.ubc.ca/~lowe/keypoints/>
- Suited for wide-baseline matching
- Invariance to changes in illumination, scale, and rotation
- No gradient approach, but compares feature description of all candidates
- Many applications

Wide-baseline Matching with SIFT

- Application example: Mosaic generation

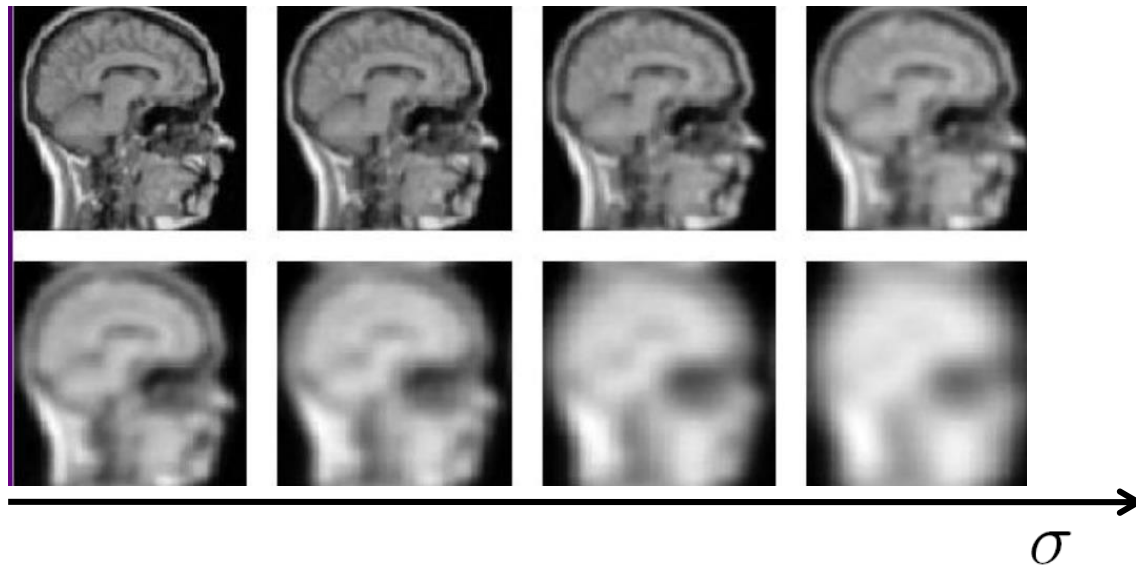


- Application example: Object recognition



- Algorithm steps
 - Scale-space extrema detection
 - Keypoint localization
 - Orientation assignment
 - Keypoint descriptor
 - Descriptor matching

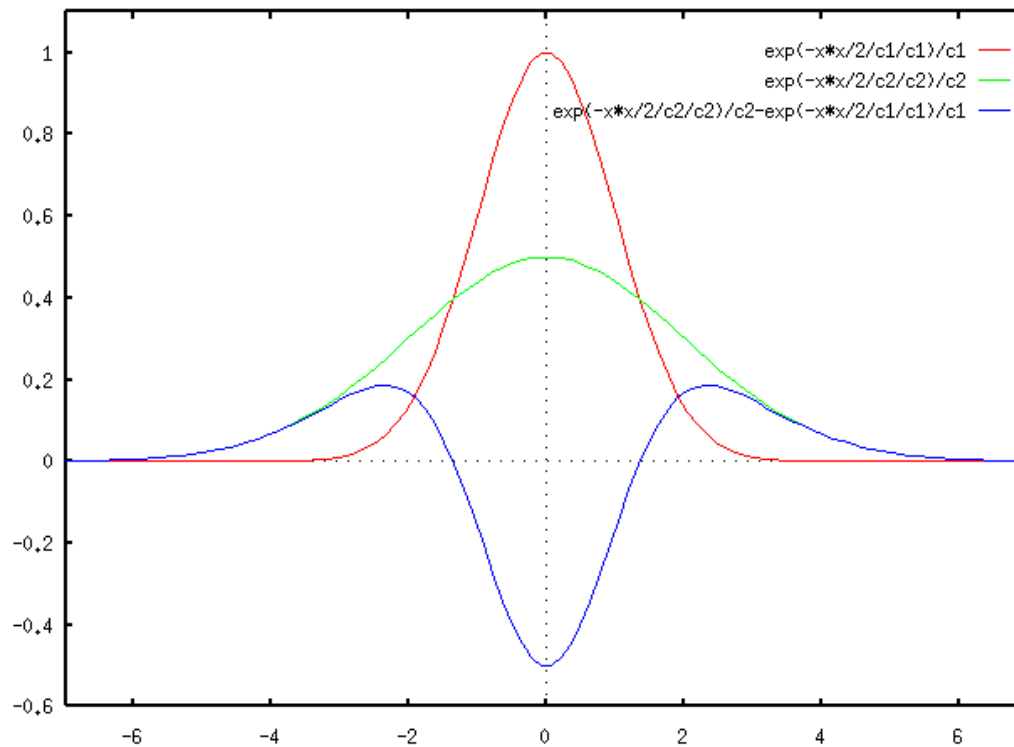
- Need to find “characteristic scale” for features
- Scale space representation



$$L(\underline{\mathbf{n}}, \sigma) = G(\underline{\mathbf{n}}, \sigma) * I(\underline{\mathbf{n}}) \quad G(\underline{\mathbf{n}}, \sigma) = \frac{1}{2\pi\sigma^2} e^{-\frac{n_x^2 + n_y^2}{2\sigma^2}}$$

■ Difference of Gaussian

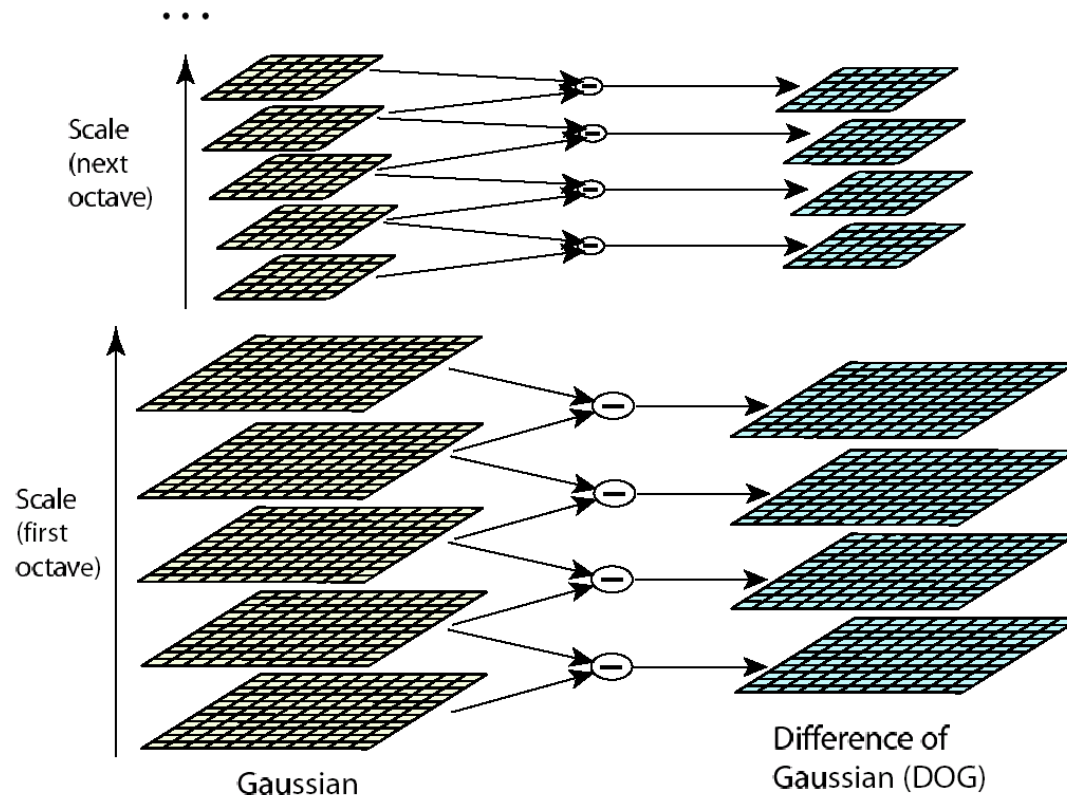
$$\text{DoG}(\underline{\mathbf{n}}) = (G(\underline{\mathbf{n}}, \sigma) - G(\underline{\mathbf{n}}, k \sigma)) * I(\underline{\mathbf{n}}) = L(\underline{\mathbf{n}}, \sigma) - L(\underline{\mathbf{n}}, k \sigma)$$



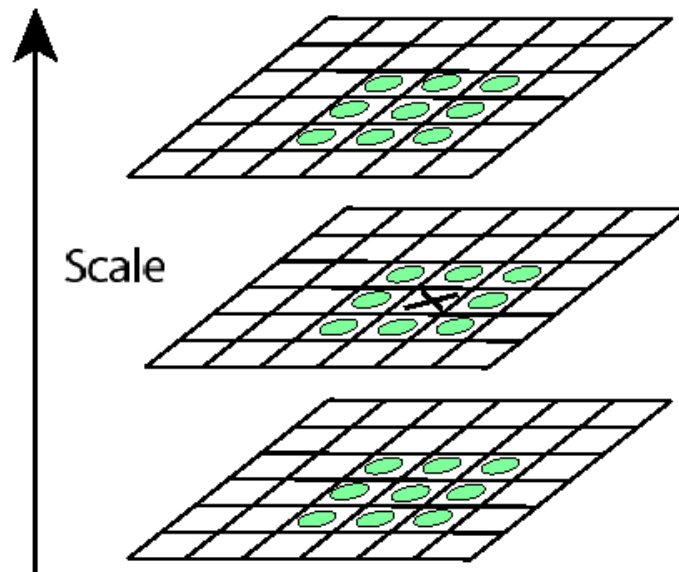
Source: <http://fourier.eng.hmc.edu/e161/lectures/gradient/node11.html>

- Difference of Gaussian

$$\text{DoG}(\underline{n}) = (G(\underline{n}, \sigma) - G(\underline{n}, k\sigma)) * I(\underline{n}) = L(\underline{n}, \sigma) - L(\underline{n}, k\sigma)$$



- Non-extremum suppression



- X is selected if it is larger or smaller than all 26 neighbors (alternatively other 3D windows)

- Sub-pixel keypoint localization

$$\text{DoG}(\underline{\mathbf{x}}) = \text{DoG}(\underline{\mathbf{n}}) + \frac{\partial \text{DoG}(\underline{\mathbf{n}})}{\partial \underline{\mathbf{x}}} \underline{\mathbf{x}} + \frac{1}{2} \underline{\mathbf{x}}^\top \frac{\partial^2 \text{DoG}(\underline{\mathbf{n}})}{\partial \underline{\mathbf{x}}^2} \underline{\mathbf{x}}$$

- Finding the extrema by setting the derivative to zero

- Contrast threshold, reject feature if

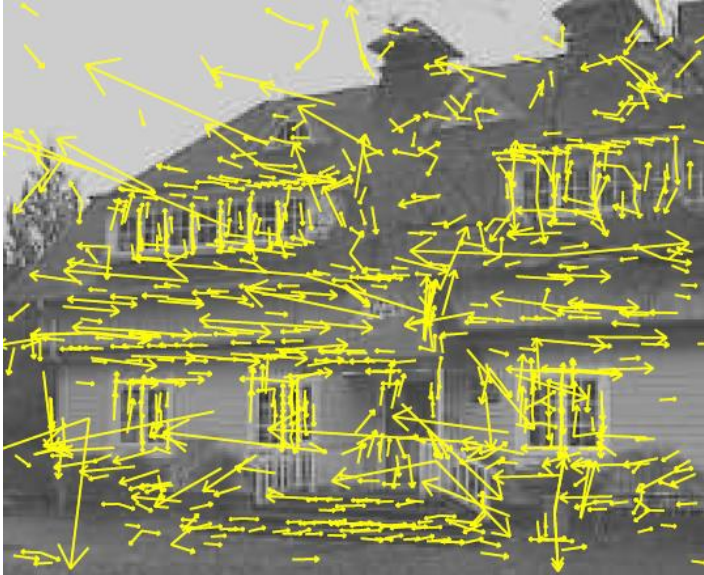
$$|\text{DoG}(\hat{\underline{\mathbf{x}}})| < 0.03$$

- Cornerness threshold: Reject edges and keep corners, reject feature if

$$\frac{\text{trace}^2(\mathbf{G}(\hat{\underline{\mathbf{x}}}))}{\det(\mathbf{G}(\hat{\underline{\mathbf{x}}}))} > \tau$$

SIFT: Keypoint localization

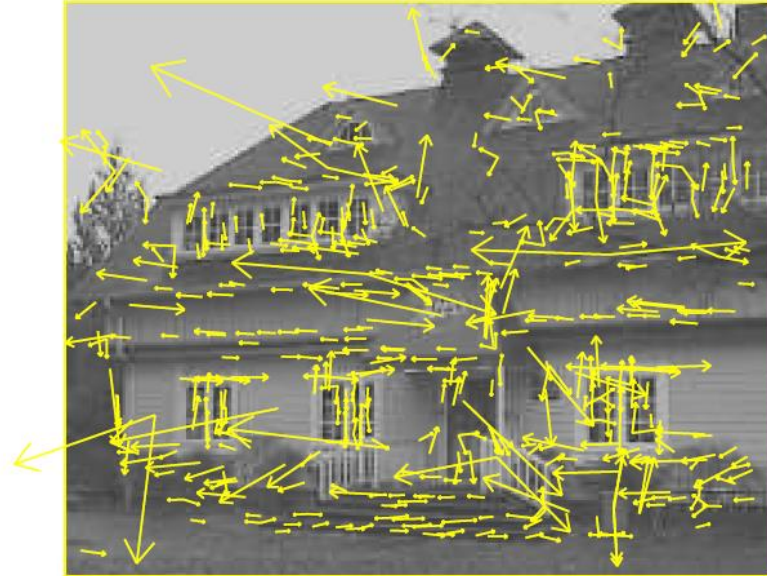
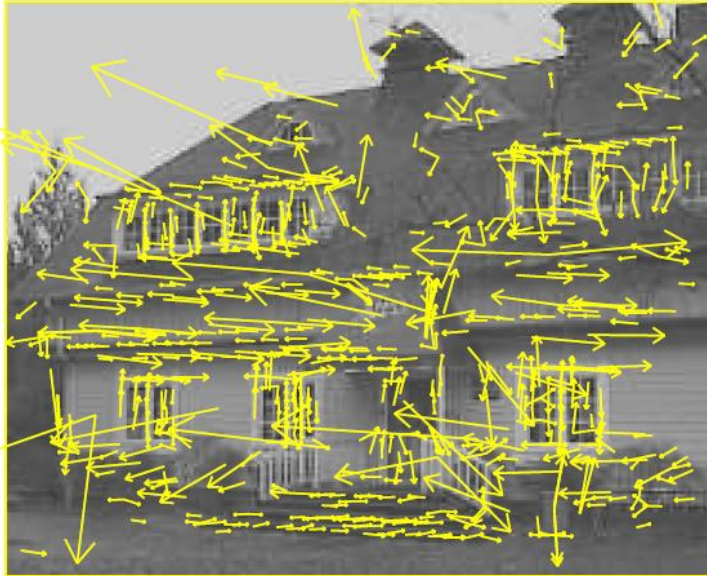
- Contrast threshold



- 729 out of 832 feature are left after contrast thresholding

SIFT: Keypoint localization

- Cornerness threshold



- 536 out of 729 are left after cornerness thresholding

- Gradient magnitude for each pixel

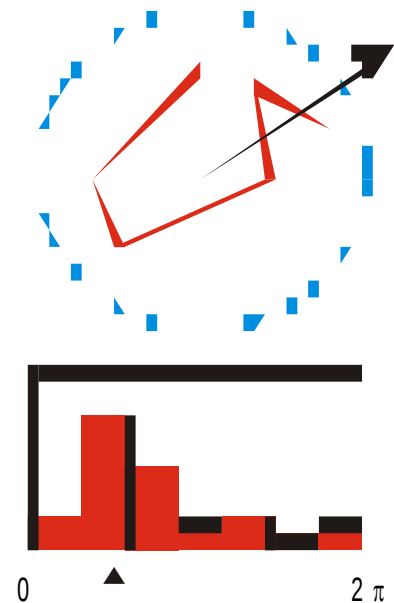
$$m(\underline{\mathbf{n}}) = \sqrt{(L(n_x + 1, n_y) - L(n_x - 1, n_y))^2 + (L(n_x, n_y + 1) - L(n_x, n_y - 1))^2}$$

- Orientation for each pixel

$$\theta(\underline{\mathbf{n}}) = \tan^{-1} \left(\frac{L(n_x, n_y + 1) - L(n_x, n_y - 1)}{L(n_x + 1, n_y) - L(n_x - 1, n_y)} \right)$$

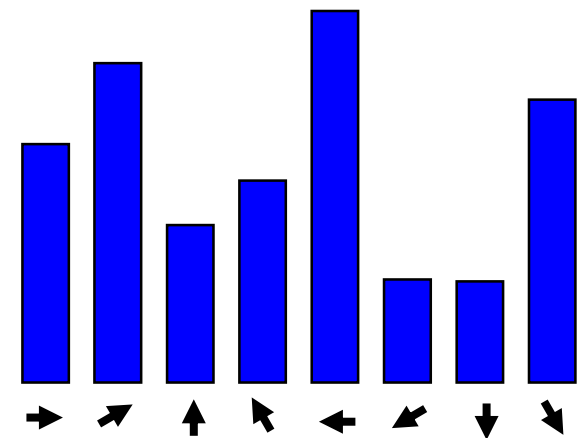
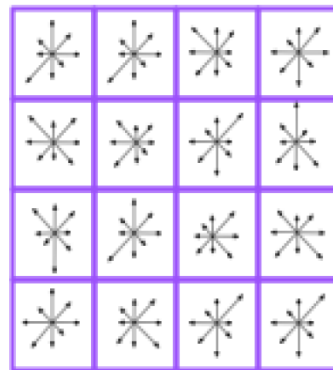
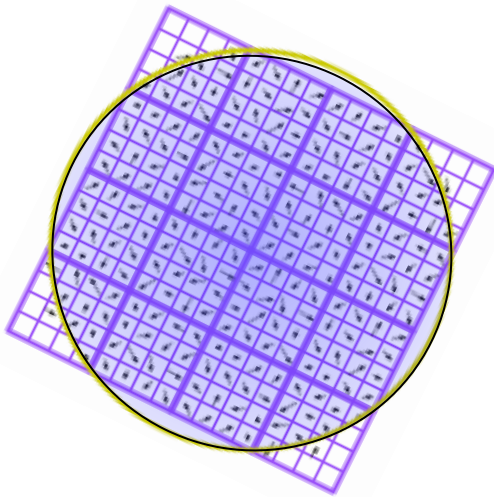
SIFT: Orientation assignment II

- Image **location**, **scale**, and **orientation** impose a repeatable local 2D coordinate system in which to describe the local image region
- A orientation histogram is formed from the gradient orientations of sample points within a region around the keypoint.
- The orientation histogram has 36 bins covering the 360 degree range of orientations.
- The highest peak in the histogram is detected



SIFT: Keypoint descriptor

- Create 16 gradient histograms (8 bins)
- Weighted by magnitude and Gaussian window (σ is half the window size)
- Histogram and gradient values are interpolated and smoothed



128-feature vector

- Nearest Neighbor algorithm based on L2-norm
- How to discard bad matches?
 - Threshold on L2-norm => bad performance
 - Solution: threshold on ratio: (best match) / (second best match)

SIFT: Descriptor matching

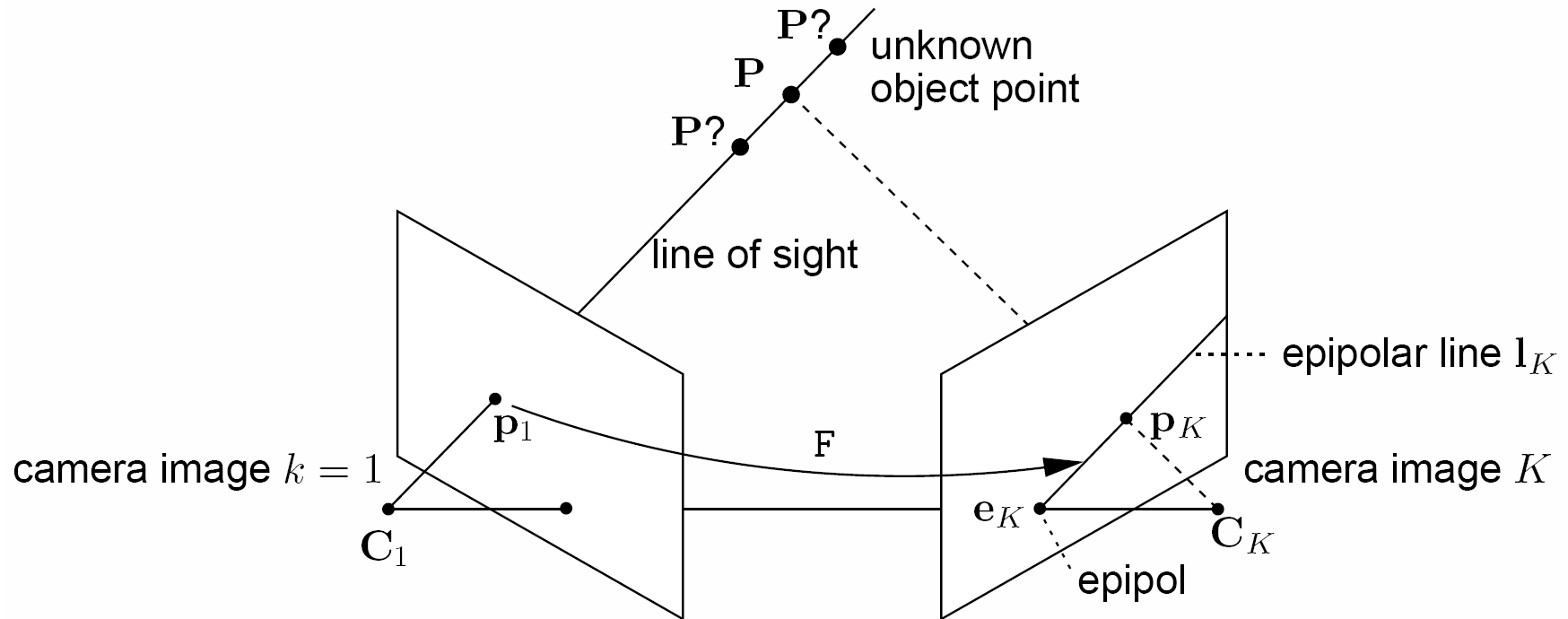


- VLFeat (<http://www.vlfeat.org/index.html>)
 - Implements SIFT and other modern features
 - C-implementation with MATLAB bindings
 - Win/Linux
 - Good tutorials on parameter selection

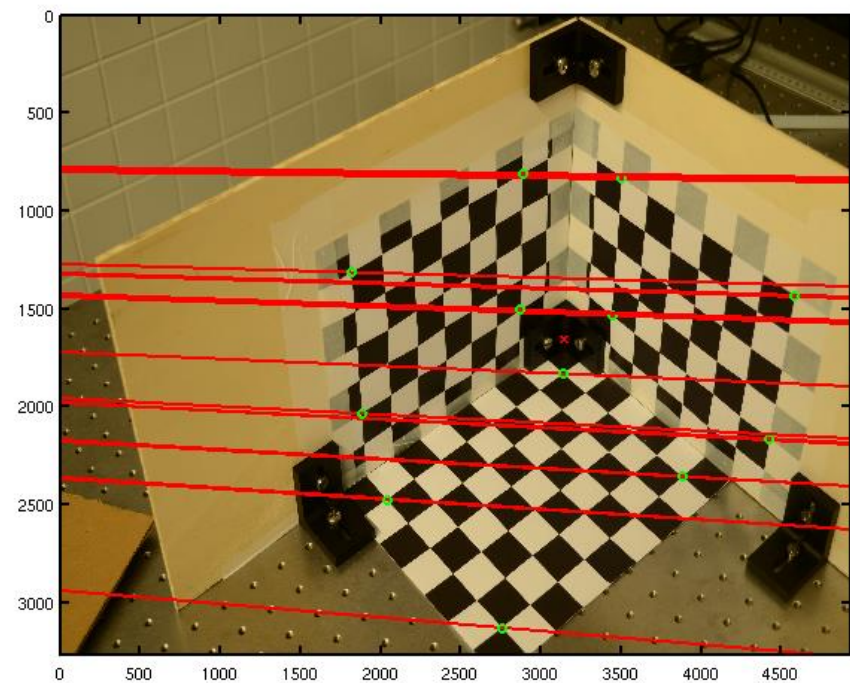
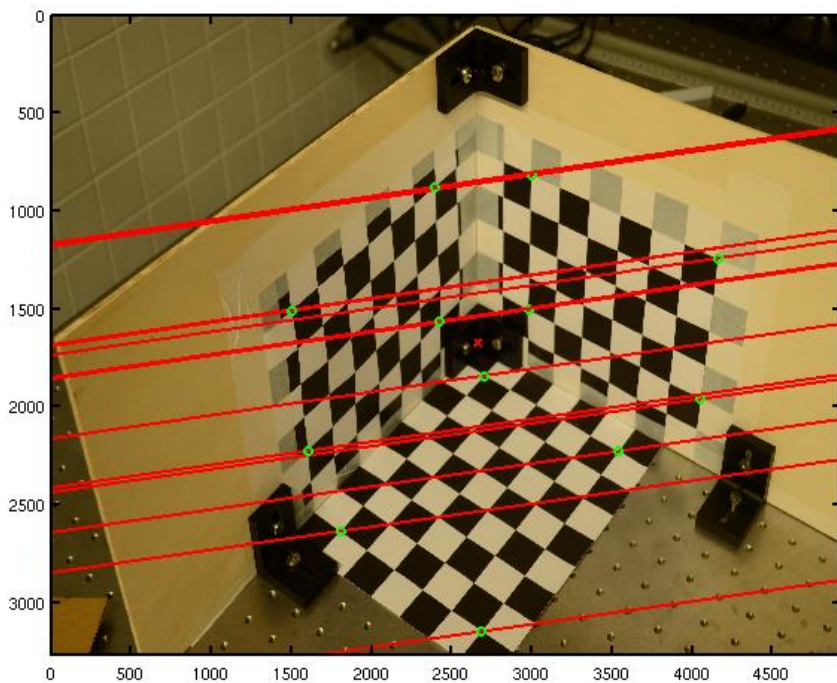
Towards Multiple Views and Self-Calibration

-- Two-View Geometry and
Basic Stereo --

Fundamental Matrix (F-Matrix): $\mathbf{p}_K^\top \mathbf{l}_K = 0 \Leftrightarrow \mathbf{p}_K^\top \mathbf{F} \mathbf{p}_1 = 0$



Epipolar Geometry in our example



- F is a rank 2 homogeneous matrix with 7 degrees of freedom

- Epipolar lines

$$\mathbf{l}_2 = F \mathbf{p}_1$$

$$\mathbf{l}_1 = F^\top \mathbf{p}_2$$

- Epipoles

$$F \mathbf{e}_1 = \mathbf{0}$$

$$F^\top \mathbf{e}_2 = \mathbf{0}$$

- F can be computed from camera matrices

- General projective cameras:

$$\mathbf{F} = [\mathbf{e}_2]_{\times} \mathbf{A}_2 \mathbf{A}_1^+$$

$$\mathbf{e}_2 = \mathbf{A}_2 \mathbf{C}_1$$

$$\mathbf{A}_1 \mathbf{C}_1 = \mathbf{0}$$

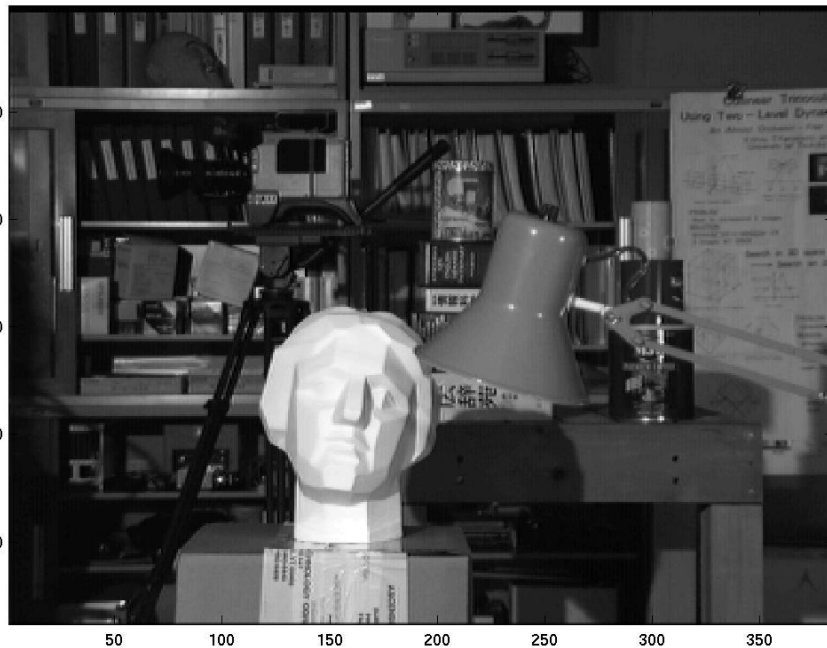
$$[\mathbf{e}_K]_{\times} = [(e_x, e_y, e_z)^{\top}]_{\times} = \begin{bmatrix} 0 & -e_z & e_y \\ e_z & 0 & -e_x \\ -e_y & e_x & 0 \end{bmatrix}$$

- Canonical cameras not at infinity $\mathbf{A}_1 = \mathbf{K}_1[\mathbf{I}|\mathbf{0}]$ and $\mathbf{A}_2 = \mathbf{K}_2[\mathbf{R}_2|\mathbf{t}_2]$

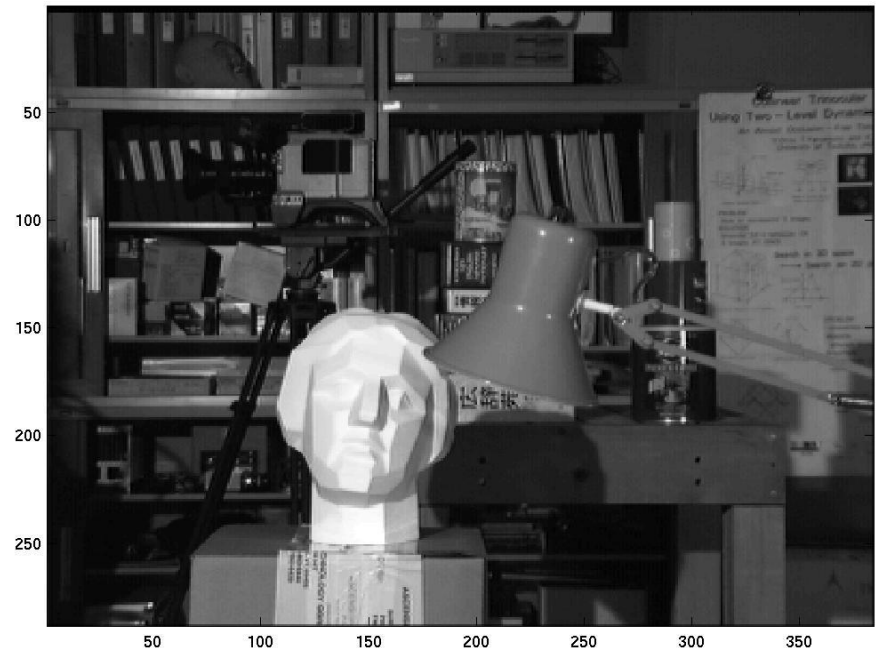
$$\mathbf{F} = \mathbf{K}_2^{-\top} [\mathbf{t}_2]_{\times} \mathbf{R}_2 \mathbf{K}_1^{-1}$$

Use Case – Stereo Matching

- Typical left and right image with parallel optical axes and only horizontally displaced



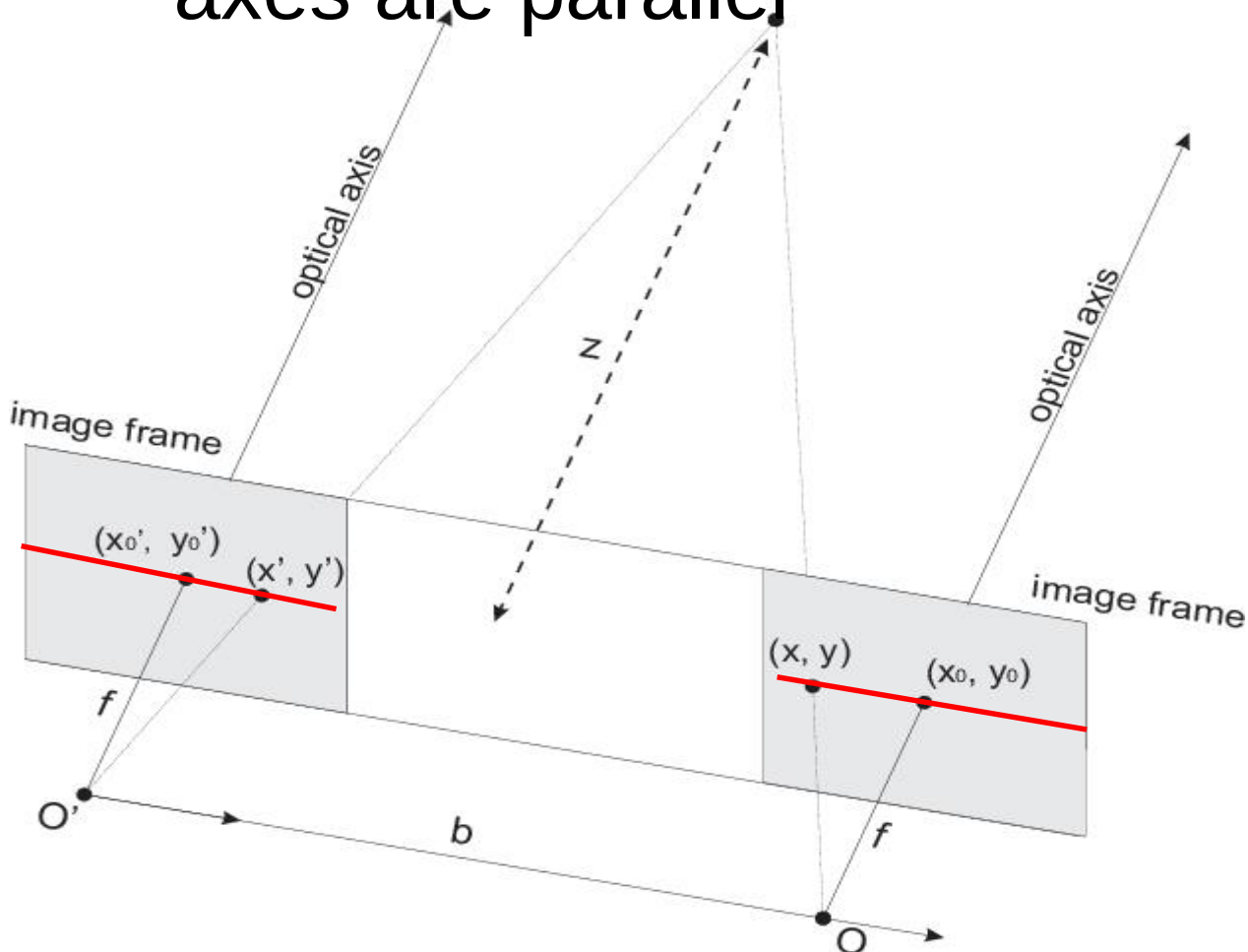
Left image



Right image

Epipolar Geometry of a Stereo Pair

- Epipolar lines are parallel lines if optical axes are parallel



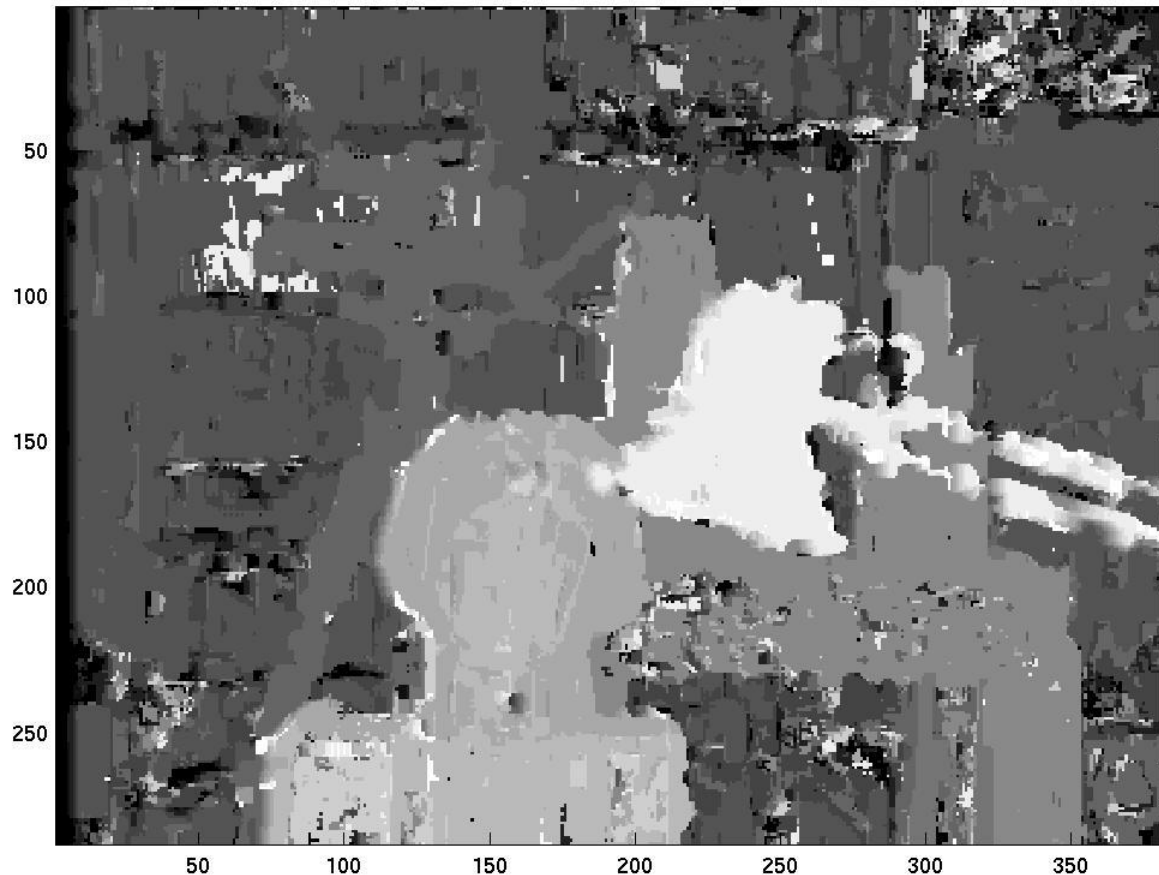
- Simplest: Search a moving window and perform correlation
- E.g. SSD (sum of squared differences)

$$\text{SSD}(\underline{\mathbf{d}}) = \sum_{m_x} \sum_{m_y} [I_K(\underline{\mathbf{n}}_{K-1} + \underline{\mathbf{d}} + \underline{\mathbf{m}}) - I_{K-1}(\underline{\mathbf{n}}_{K-1} + \underline{\mathbf{m}})]^2 \rightarrow \min$$

$$\underline{\mathbf{d}} = s \cdot \begin{pmatrix} d_x \\ d_y \end{pmatrix} \text{ with } s \in \mathbb{R} \text{ and } \begin{pmatrix} d_x \\ d_y \end{pmatrix} \text{ const.}$$

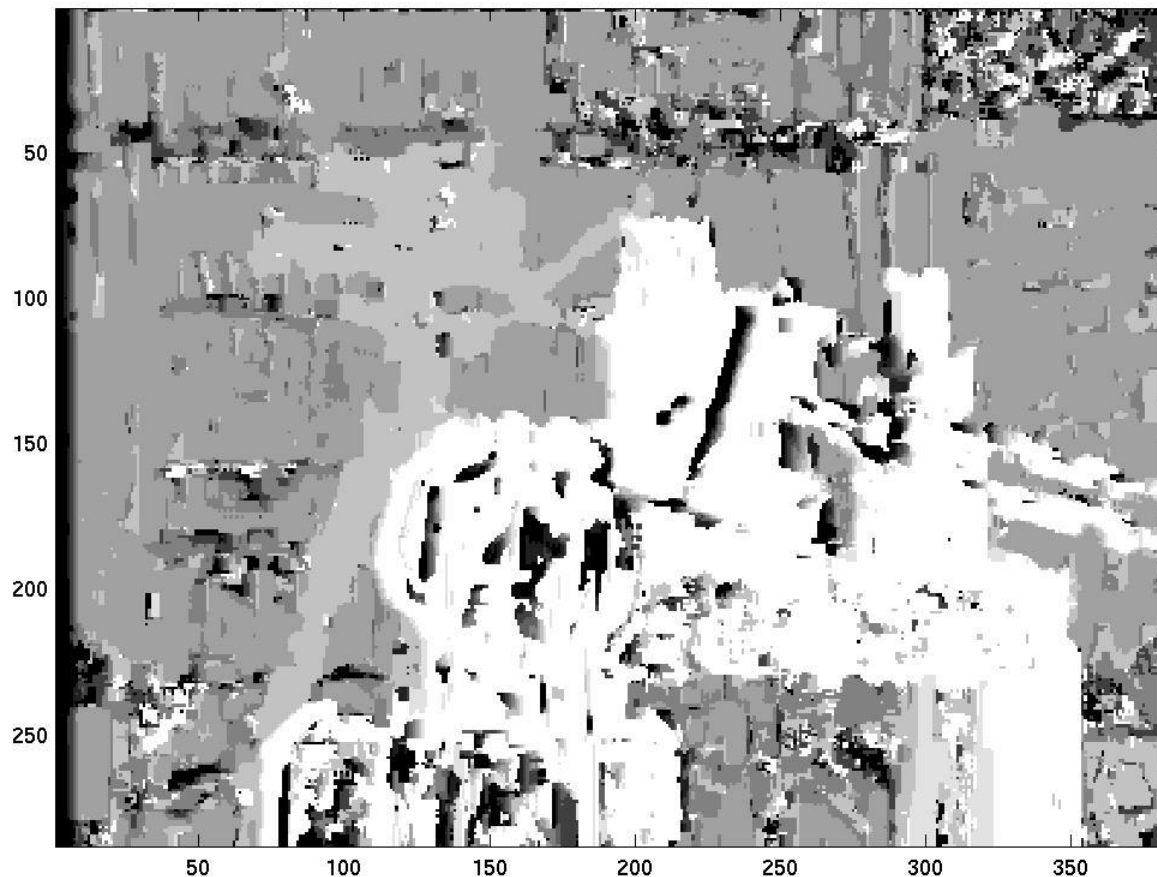
- Alternatively, SAD (sum of absolute differences), CC (cross correlation), etc.
- Usually restricted search range

- Example, SSD, win=5, search range=15

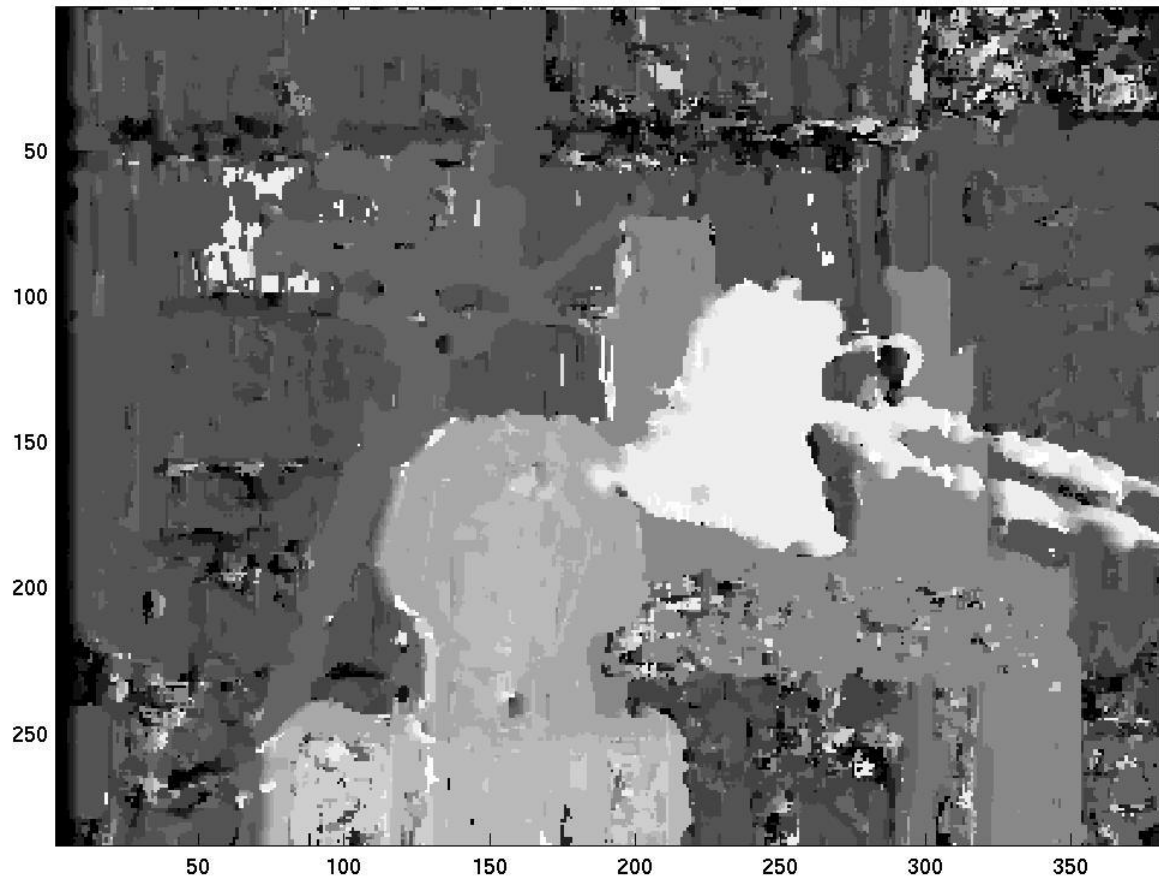


Correlation-Based Stereo Matching

- Example, SSD, win=5, search range=8

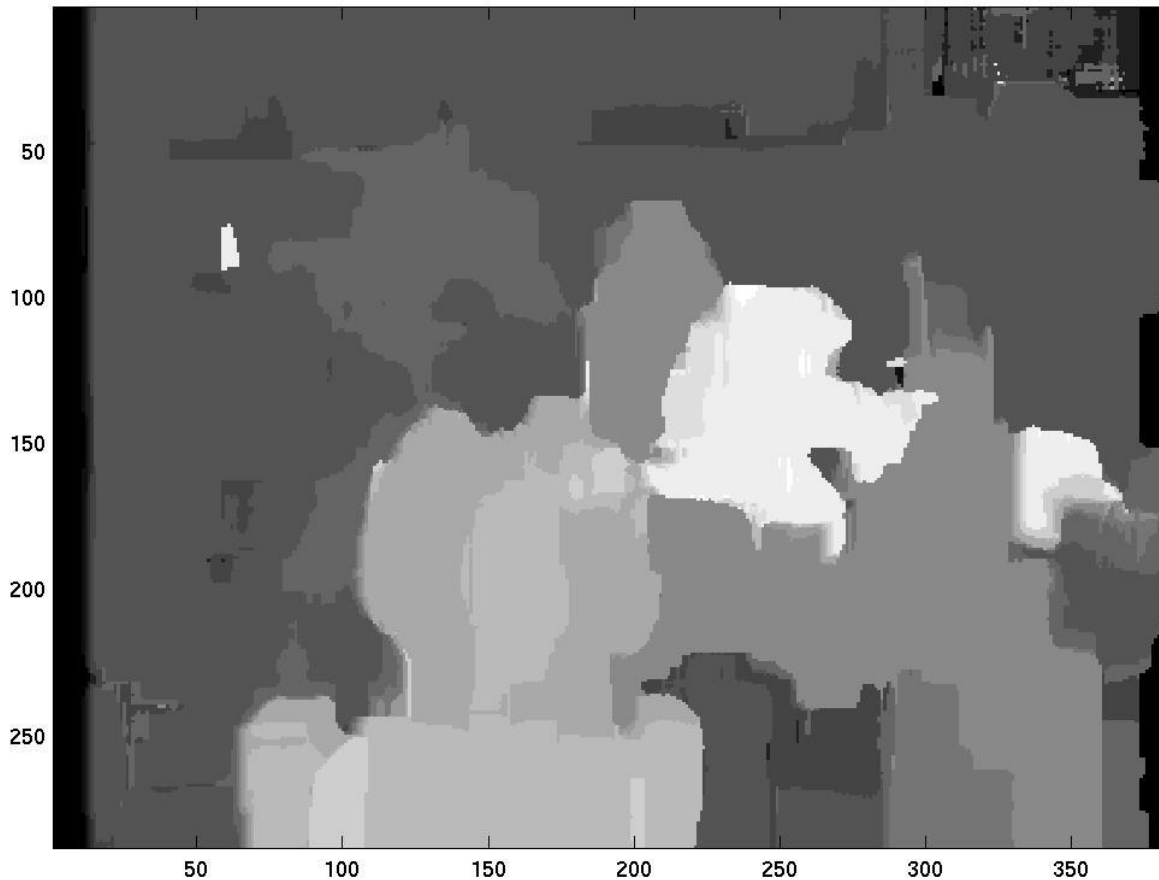


- Example, **SAD**, win=5, search range=8



Correlation-Based Stereo Matching

- Example, SSD, win=**20**, search range=8



A Modern Technique

- ADCensus [Mei'11] (#1 Middlebury stereo benchmark, Sept. 2013)

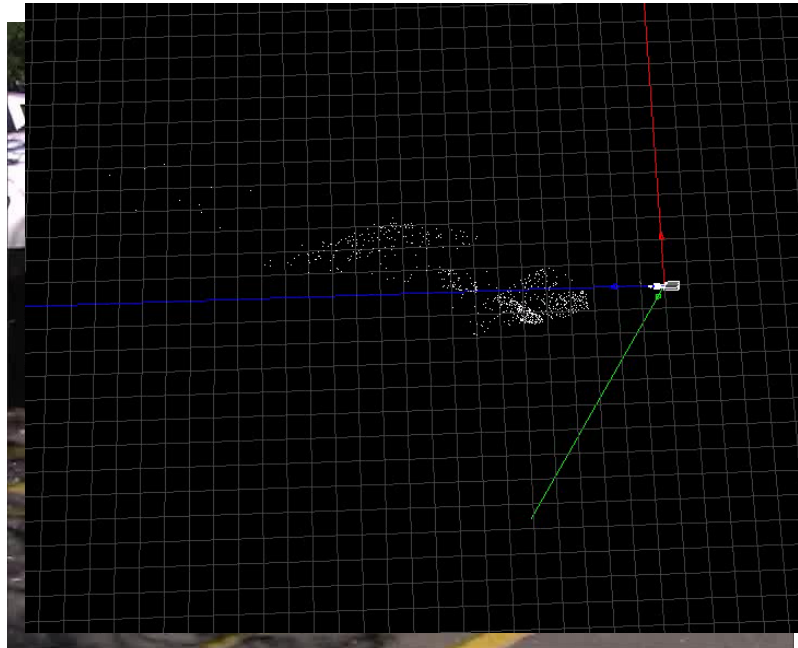
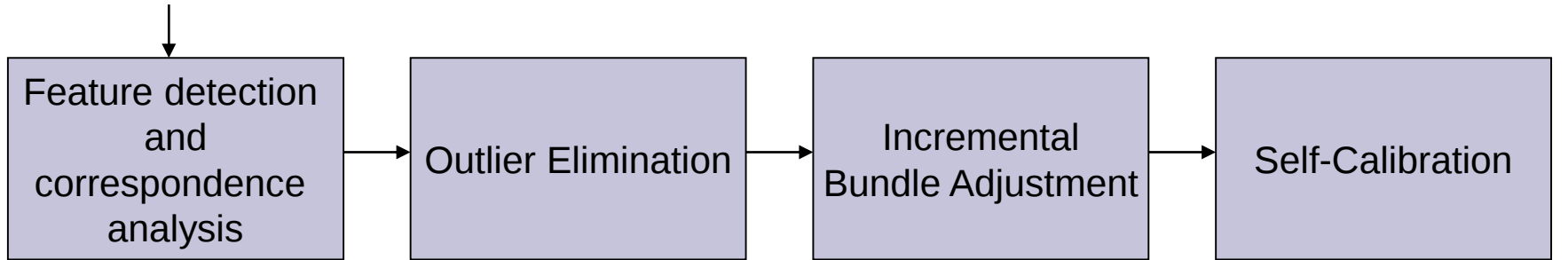


Towards Multiple Views and Self-Calibration

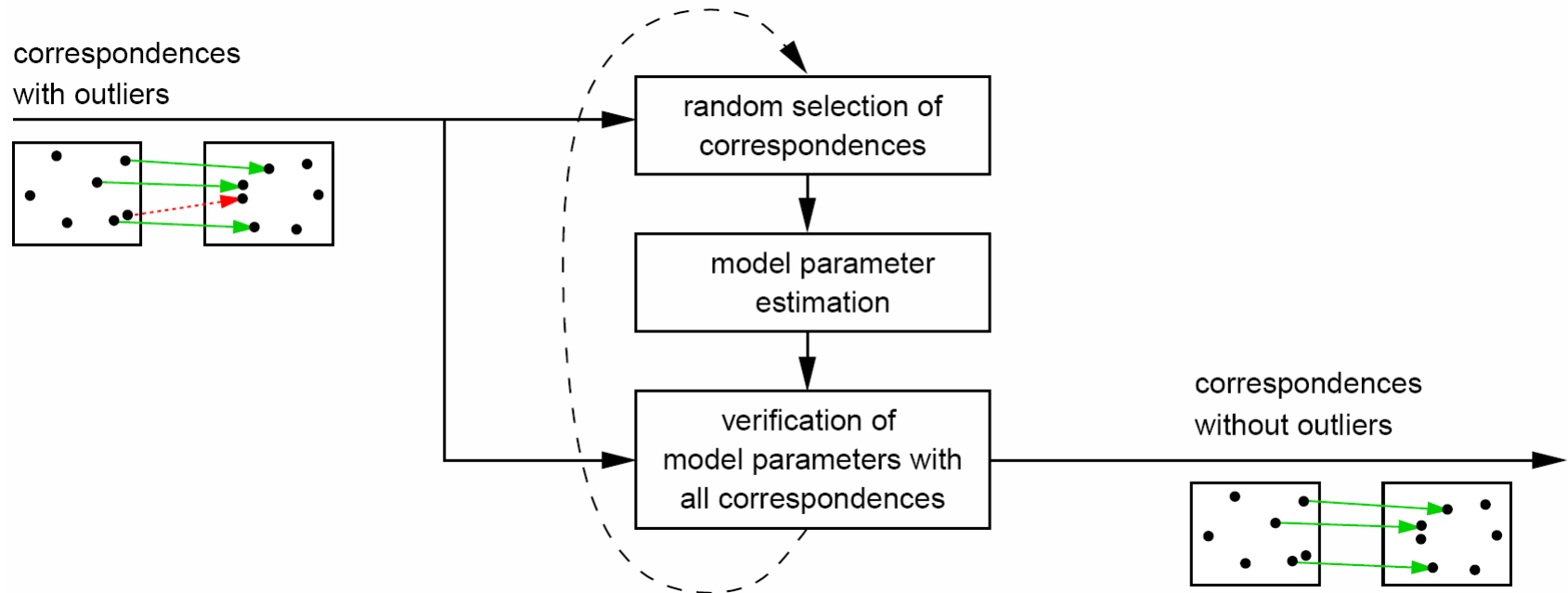
-- The Structure-from-motion
Pipeline --

Voodoo Camera Tracker - Steps of camera tracking

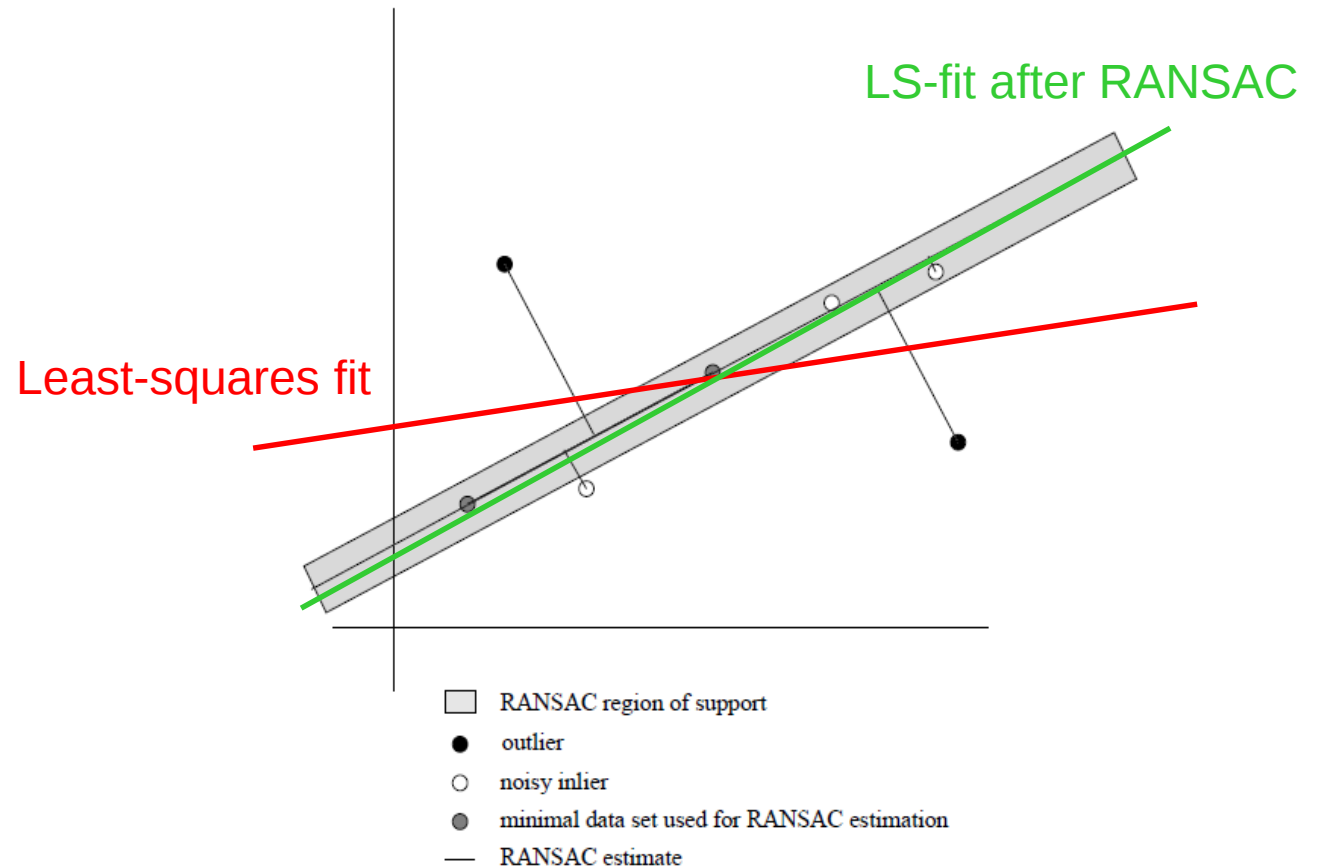
image sequence



- RANSAC (Random Sample Consensus) method

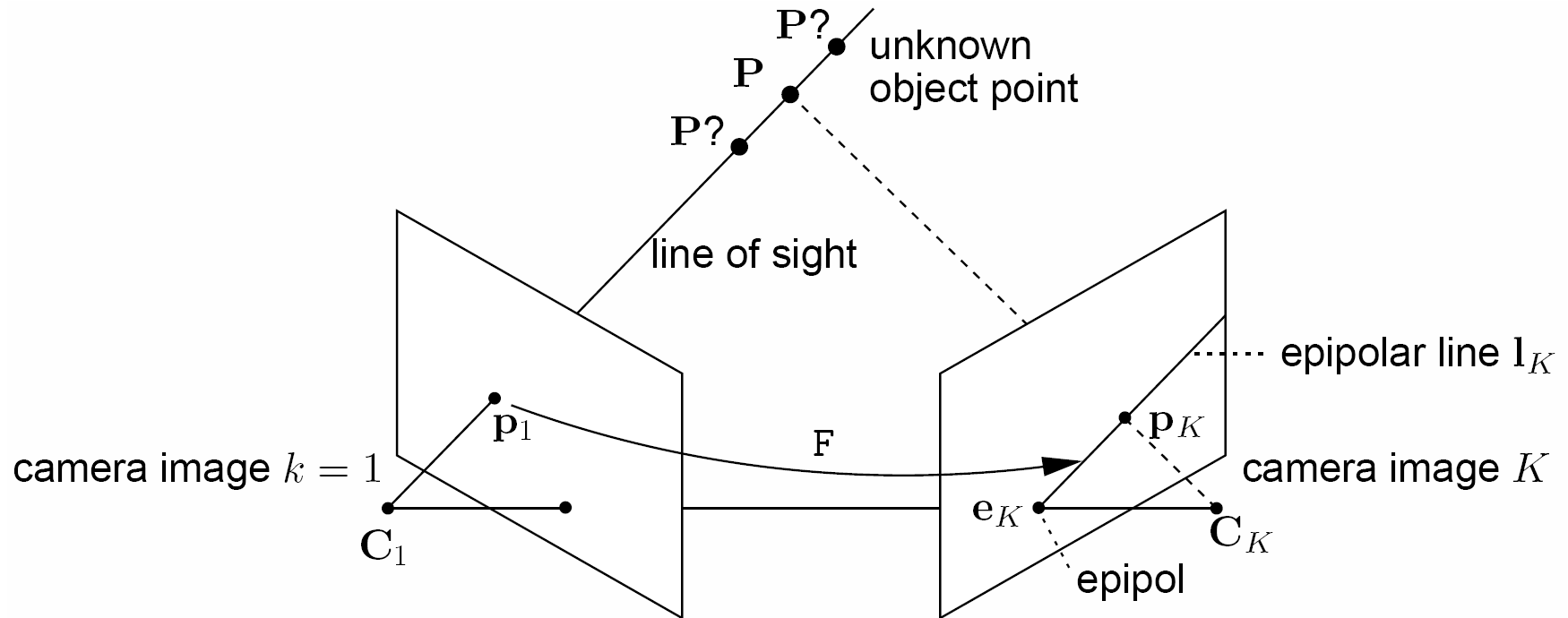


RANSAC example – line fit



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Fundamental Matrix (F-Matrix): $\mathbf{p}_K^\top \mathbf{l}_K = 0 \quad \Leftrightarrow \quad \mathbf{p}_K^\top \mathbf{F} \mathbf{p}_1 = 0$



Fundamental Matrix Estimation

Estimating the fundamental matrix from
feature correspondence:

$$\mathbf{p}_2^T \mathbf{F} \mathbf{p}_1 = 0$$

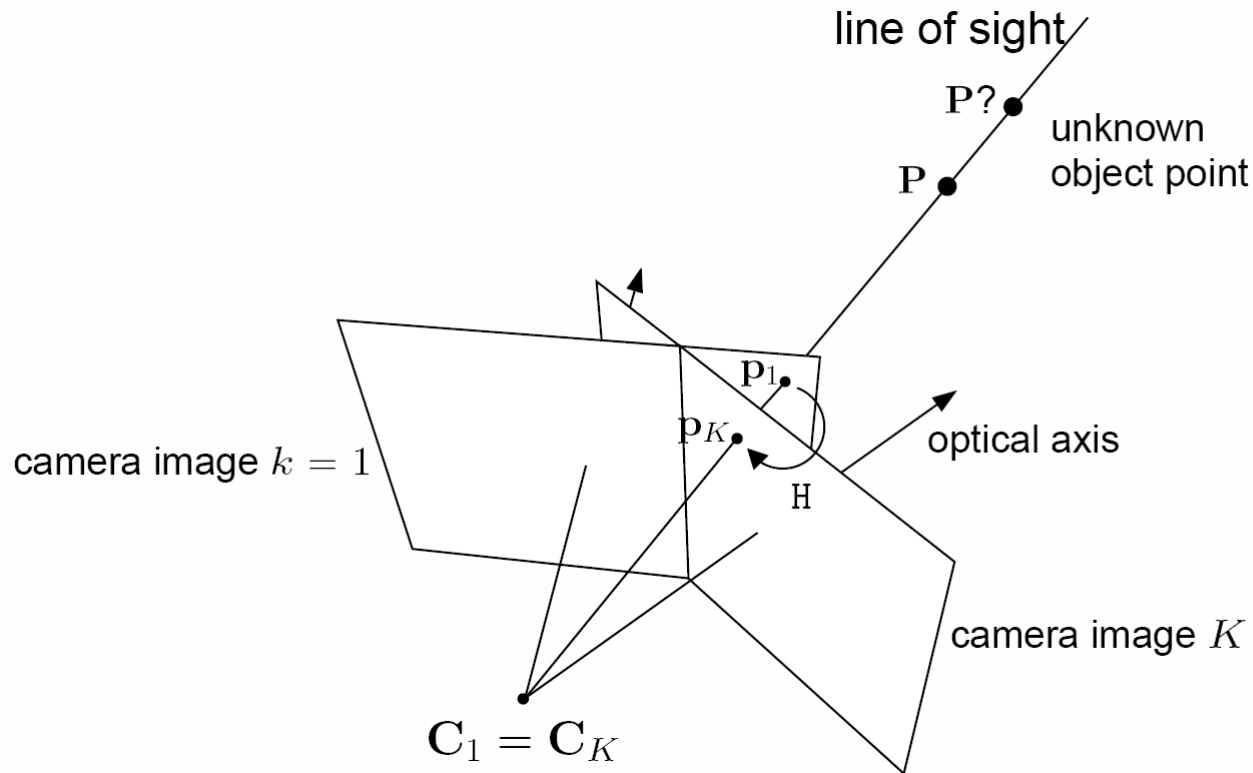
$$(x_2, y_2, 1) \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} = 0$$

$$\begin{pmatrix} x_2 x_1 & x_2 y_1 & x_2 & y_2 x_1 & y_2 y_1 & y_2 & x_1 & y_1 & 1 \end{pmatrix} \begin{pmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{pmatrix} = 0$$

multiple of these equations gives a linear equation system $\mathbf{A} \mathbf{f} = \mathbf{0}$

Outlier Elimination - 2D Homography

2D Homography (H-Matrix) $\mathbf{p}_K = \mathbf{H} \mathbf{p}_1$



2D Homography Matrix Estimation

Estimating the 2D homography matrix
from feature correspondence:

$$\mathbf{H} \mathbf{p}_1 = \mathbf{p}_2$$

$$\mathbf{p}_2 \times \mathbf{H} \mathbf{p}_1 = \mathbf{p}_2 \times \mathbf{p}_2$$

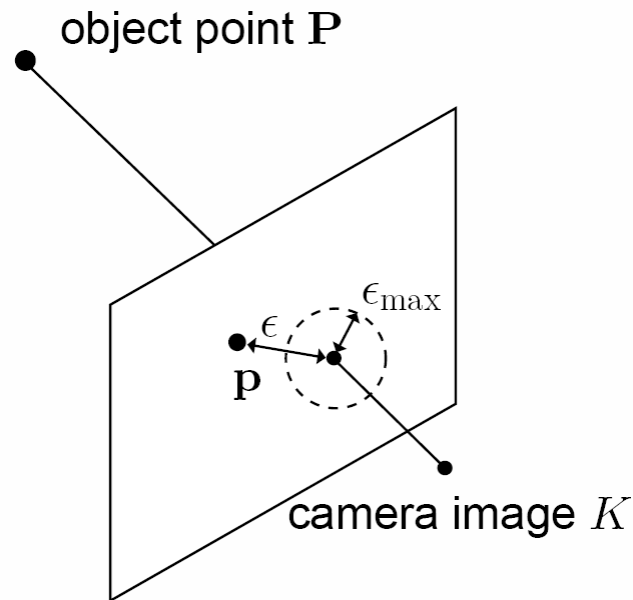
$$\mathbf{p}_2 \times \mathbf{H} \mathbf{p}_1 = \mathbf{0}$$

$$\begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix} \times \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix} \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} = \mathbf{0}$$

$$\begin{pmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_2x_1 & -x_2y_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -y_2x_1 & -y_2y_1 \end{pmatrix} \begin{pmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \end{pmatrix} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$$

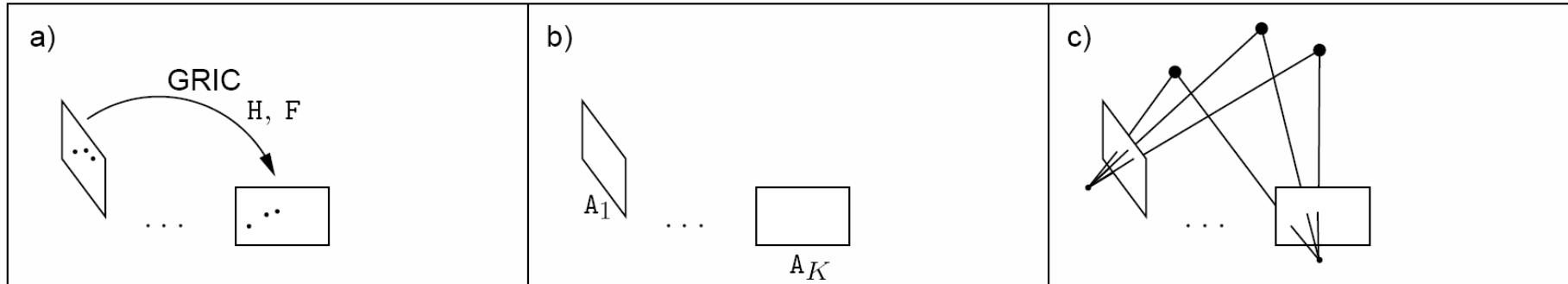
multiple of these equations gives a linear equation system $\mathbf{A} \mathbf{h} = \mathbf{b}$

Camera matrix (A-Matrix): $\mathbf{p}_{j, K} = \mathbf{A}_K \mathbf{P}_j$

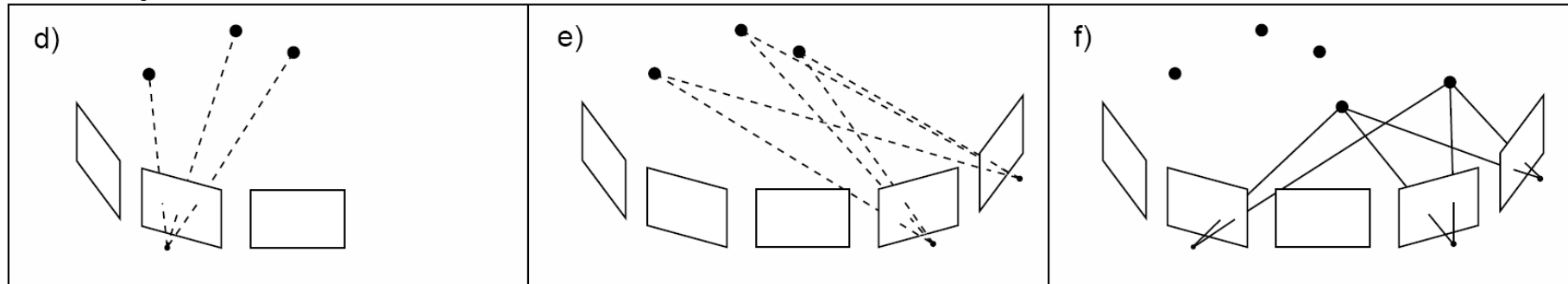


Incremental Bundle Adjustment

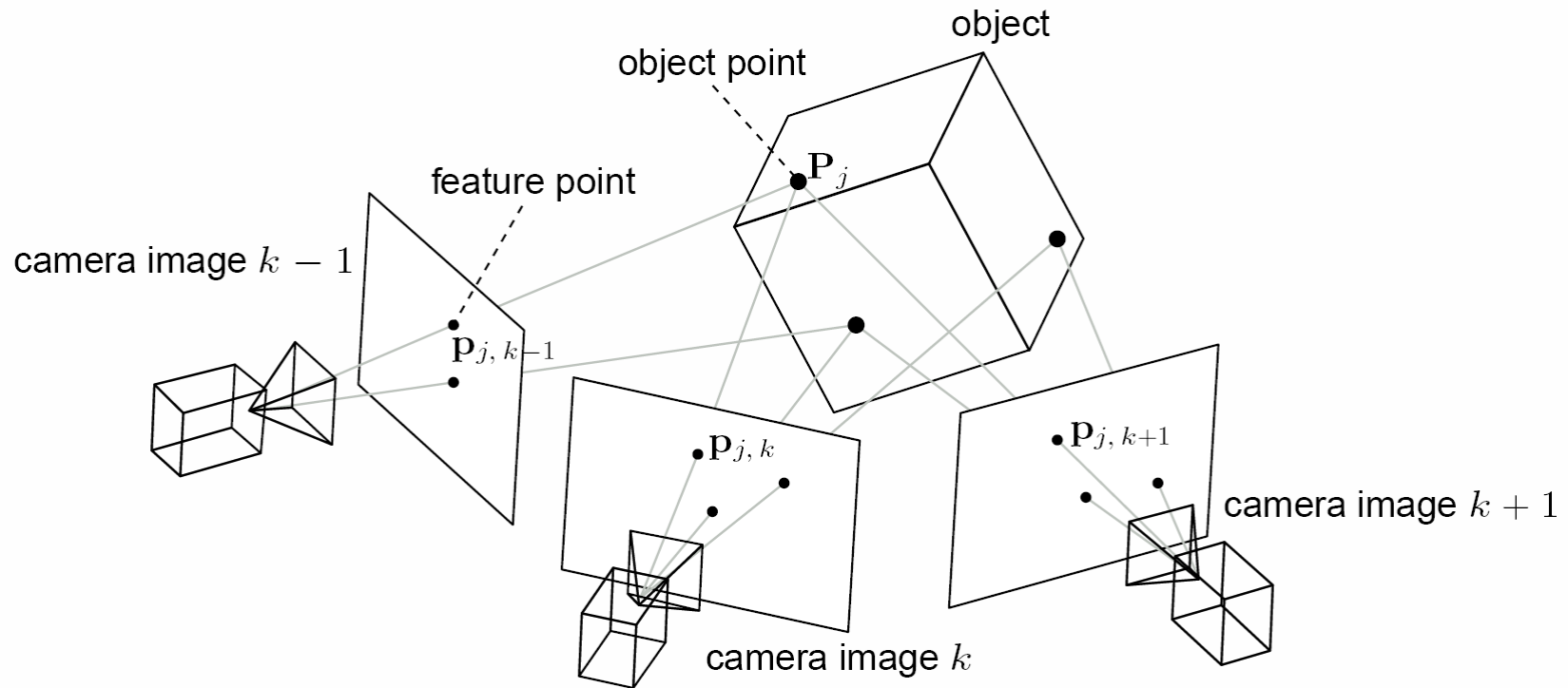
Initialization State:



3D Projection State:



Bundle Adjustment



$$\arg \min_{\hat{\mathbf{A}}_k, \hat{\mathbf{P}}_j} \sum_k \sum_j d(\tilde{\mathbf{p}}_{j, k}, \hat{\mathbf{A}}_k \hat{\mathbf{P}}_j)^2$$

Levenberg Marquardt - Non-linear Least Squares

$$\arg \min_{\hat{\mathbf{X}}} \left(\tilde{\mathbf{Y}} - f(\hat{\mathbf{X}}) \right)^2$$

$\tilde{\mathbf{Y}} \in \mathbb{R}^N$ Measurement vector

$\hat{\mathbf{X}} \in \mathbb{R}^M$ Parameter vector

Taylor approximation:

$$f(\hat{\mathbf{X}}) = f(\mathbf{X}_0 + \Delta_{\hat{\mathbf{X}}}) \approx f(\mathbf{X}_0) + \mathbf{J} \Delta_{\hat{\mathbf{X}}}$$

with $N \times M$ Jacobian Matrix

$$\mathbf{J} = \frac{\partial \tilde{\mathbf{Y}}}{\partial \hat{\mathbf{X}}}$$

Levenberg Marquardt - Non-linear Least Squares

$$\begin{aligned} & \arg \min_{\hat{\mathbf{X}}} \left(\tilde{\mathbf{Y}} - f(\hat{\mathbf{X}}) \right)^2 \\ &= \arg \min_{\Delta_{\hat{\mathbf{X}}}} \left(\tilde{\mathbf{Y}} - f(\mathbf{X}_0 + \Delta_{\hat{\mathbf{X}}}) \right)^2 \\ &\approx \arg \min_{\Delta_{\hat{\mathbf{X}}}} \left(\tilde{\mathbf{Y}} - (f(\mathbf{X}_0) + \mathbf{J} \Delta_{\hat{\mathbf{X}}}) \right)^2 \\ &= \arg \min_{\Delta_{\hat{\mathbf{X}}}} \left((\tilde{\mathbf{Y}} - f(\mathbf{X}_0)) - \mathbf{J} \Delta_{\hat{\mathbf{X}}} \right)^2 \\ &= \arg \min_{\Delta_{\hat{\mathbf{X}}}} (\epsilon_0 - \mathbf{J} \Delta_{\hat{\mathbf{X}}})^2 \end{aligned}$$

transformed to linear least squares problem for each iteration

$$\hat{\mathbf{X}}_1 = \hat{\mathbf{X}}_0 + \Delta_{\hat{\mathbf{X}}} \quad , \quad \hat{\mathbf{X}}_2 = \hat{\mathbf{X}}_1 + \Delta_{\hat{\mathbf{X}}} \quad , \quad \dots$$

Levenberg Marquardt - Non-linear Least Squares

Linear least squares problem can be solved with normal equations

$$\epsilon_0 - J\Delta_{\hat{\mathbf{X}}} = \mathbf{0}$$

$$J\Delta_{\hat{\mathbf{X}}} = \epsilon_0$$

$$J^T J\Delta_{\hat{\mathbf{X}}} = J^T \epsilon_0$$

$$\Delta_{\hat{\mathbf{X}}} = (J^T J)^{-1} J^T \epsilon_0$$

use to update solution iteratively

$$\hat{\mathbf{X}}_1 = \hat{\mathbf{X}}_0 + \Delta_{\hat{\mathbf{X}}} \quad , \quad \hat{\mathbf{X}}_2 = \hat{\mathbf{X}}_1 + \Delta_{\hat{\mathbf{X}}} \quad , \quad \dots$$

Levenberg Marquardt - Non-linear Least Squares

- Levenberg Marquardt uses slightly different normal equations

Original normal equations

$$J^T J \Delta_{\hat{\mathbf{x}}} = J^T \epsilon_0$$

Modified normal equations

$$[J^T J + \lambda \text{diag}(J^T J)] \Delta_{\hat{\mathbf{x}}} = J^T \epsilon_0$$

- Lambda is changed during optimization

$$\lambda_{i+1} = 0.1 \lambda_i \quad \text{if } \epsilon_i < \epsilon_{i-1} \quad \text{successful iteration}$$

$$\lambda_{i+1} = 10.0 \lambda_i \quad \text{if } \epsilon_i > \epsilon_{i-1} \quad \text{failed iteration}$$

small λ ~ Newton style (quadratic convergence)

large λ ~ Gradient descent style (guaranteed decrease)

Requirements for Levenberg Marquardt minimization

- Function to compute f
- Start value X_0
- Optionally, function to compute J
(but numerical derivation works as well)

- For bundle adjustment the problem becomes to large
(100 cameras + 10000 3D object points = 31200 parameter)
- Can achieve huge speed-up by exploiting sparse structure of Jacobian matrix
- Partition parameters
 - partition A
 - partition B (only dependent on A and itself)

(typically A contains camera parameters, and B contains 3D points)

Jacobian becomes

$$J = [A|B] \quad J \Delta_{\hat{\mathbf{x}}} = \epsilon$$
$$[A|B] \begin{pmatrix} \delta_a \\ \delta_b \end{pmatrix} = \epsilon$$

Normal equations

$$J^T J \Delta_{\hat{\mathbf{x}}} = J^T \epsilon_0$$

become

$$\begin{bmatrix} A^T A & A^T B \\ B^T A & B^T B \end{bmatrix} \begin{pmatrix} \delta_a \\ \delta_b \end{pmatrix} = \begin{pmatrix} A^T \epsilon \\ B^T \epsilon \end{pmatrix}$$

Corresponding block structure is

$$\begin{bmatrix} U^* & W \\ W^\top & V^* \end{bmatrix} \begin{pmatrix} \delta_a \\ \delta_b \end{pmatrix} = \begin{pmatrix} \epsilon_A \\ \epsilon_B \end{pmatrix}$$

where U^* denotes U augmented by multiplying its diagonal entries by a factor of $1 + \lambda$ and V^* likewise. Left multiplication with
yields

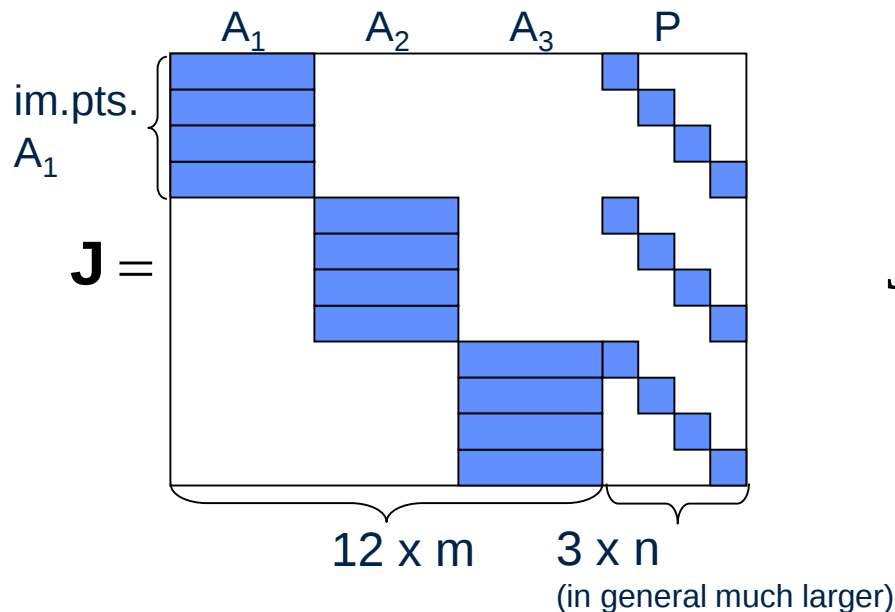
$$\begin{bmatrix} I & -WV^{*-1} \\ 0 & I \end{bmatrix}$$

$$\begin{bmatrix} U^* - WV^{*-1}W^\top & 0 \\ W^\top & V^* \end{bmatrix} \begin{pmatrix} \delta_a \\ \delta_b \end{pmatrix} = \begin{pmatrix} \epsilon_A - WV^{*-1}\epsilon_B \\ \epsilon_B \end{pmatrix}$$

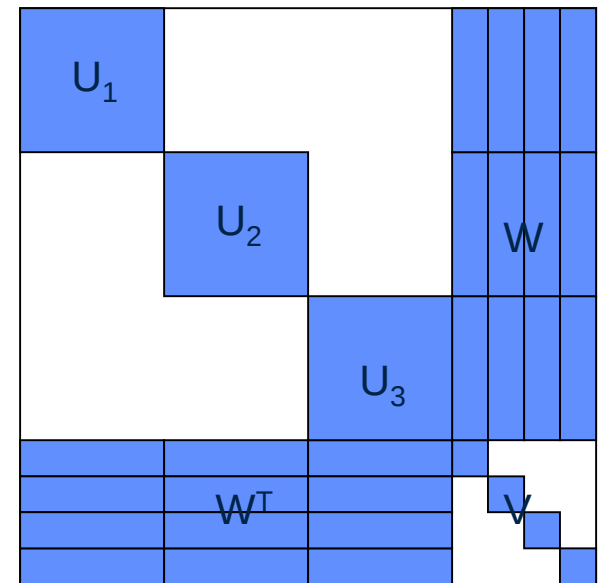
which can be used to find δ_a with $(U^* - WV^{*-1}W^\top) \delta_a = \epsilon_A - WV^{*-1}\epsilon_B$

which may be back-substituted to get δ_b with $V^* \delta_b = \epsilon_B - W^\top \delta_a$

Jacobian for bundle adjustment has sparse block structure



$$\mathbf{J}^T \mathbf{J} =$$



Needed for non-linear minimization

Voodoo camera tracker – demo session



- Why self-calibration?
 - Allows flexible acquisition
 - No prior calibration necessary
 - Possibility to vary intrinsic camera parameters
 - Use archive footage

- We want to find a 4x4 transformation matrix T_M that transforms all projective cameras into metric cameras $\check{A}_k$

$$\check{A}_k = A_k T_M \quad \forall \quad k$$

$$\check{\mathbf{P}}_j = T_M^{-1} \mathbf{P}_j \quad \forall \quad j$$

- This does not change the back-projections onto the feature points

$$\mathbf{p}_{j,k} = \check{A}_k \check{\mathbf{P}}_j = A_k T_M T_M^{-1} \mathbf{P}_j = A_k \mathbf{P}_j$$

Voodoo Camera Tracker - Some applications

- Virtual advertising

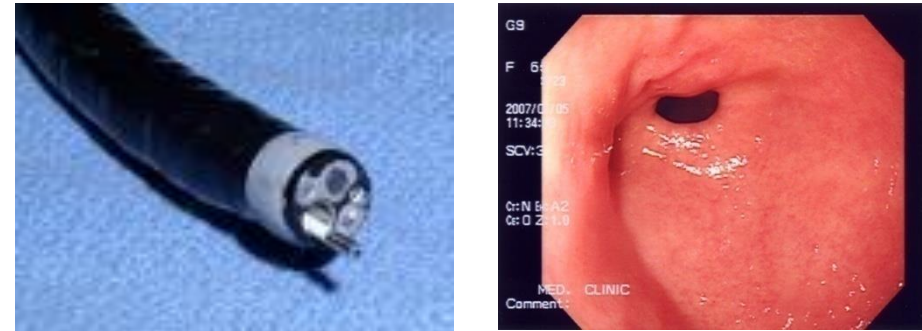


Voodoo Camera Tracker - More applications

■ Car navigation



■ 3D Endoscopy



■ UAV terrain reconstruction



■ Architectural visualisation



Matchmoving in Cloverfield



Matchmoving in Cloverfield



1. Multiple View Geometry in Computer Vision, Richard Hartley and Andrew Zisserman, 2nd Edition
2. Triggs, B.: “Autocalibration and the absolute quadric”. In “IEEE Conference on Computer Vision and Pattern Recognition”, S. 609–614. 1997
3. Pollefeys, M., Koch, R., Gool, L. V.: “Self-Calibration and Metric Reconstruction in Spite of Varying and Unknown Internal Camera Parameters”. In “EEE International Conference on Computer Vision, S. 90–95. 1998

Homework: Camera motion estimation with Voodoo

- Download and install voodoo camera tracker 1.0.1 from course website or from

<http://www.digilab.uni-hannover.de/download.html>

- Download and install Blender 2.49b from course website or from

<http://www.blender.org/download/get-blender/>

(maybe you also need to install Python)

Homework: Camera motion estimation with Voodoo

- Perform camera tracking with voodoo using the „free move“ example sequence
- Export data File → Save → Blender Python Script
- Load the python script into Blender's text editor and execute the script with ALT-P.
- Select the "voodoo_render_cam" camera and press Ctrl-Numpad 0 to activate it.
- To display the image sequence as a backbuffer in Blender, go to [Buttons

Homework: Camera motion estimation with Voodoo

- Add a (animated) 3D model of your choice
- Render the sequence
- Submit frame 190, 200, and 210 as jpeg to thormae@mpii.de
- Submit before next lecture
- (optional) generate an avi-file and put it to a webspace that is accessible for me and send the link

That's it for today



Source: Belfast Telegraph, UK